Math 49B
Quiz 3
Fri Jan 25, 2008

Code: ALL COLORS
I forgot my code, so please charge me two points: (Name) ____________

[4 POINTS] Use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \frac{\pi}{2}$.

$\sqrt{x^2 - 9}, \quad x = 3\sec\theta$

$$\sqrt{(3\sec\theta)^2 - 9}$$
$$= \sqrt{9\sec^2\theta - 9}$$
$$= \sqrt{9(\sec^2\theta - 1)}$$
$$= \sqrt{9\tan^2\theta}$$
$$= 3\tan\theta$$

1 POINT EACH

[6 POINTS] Verify the identity $\sin y \tan y = \sec y - \cos y$.

$$\sec y - \cos y$$
$$= \frac{1}{\cos y} - \cos y \quad 1$$
$$= \frac{1 - \cos^2 y}{\cos y} \quad 2$$
$$= \frac{\sin^2 y}{\cos y} \quad 1$$
$$= \sin y \frac{\sin y}{\cos y} \quad 1$$
$$= \sin y \tan y \quad 1$$

[QUESTIONS ON OTHER SIDE]

[4 POINTS] Factor the expression and use the fundamental identities to simplify.

$$1-2\csc^2 t+\csc^4 t$$

$$=(\csc^2 t-1)^2 \quad 2$$

$$=(\cot^2 t)^2 \quad 1$$

$$=\cot^4 t \quad 1$$

[6 POINTS] Verify the identity $\dfrac{1}{1+\sin x}+\dfrac{1}{1+\csc x}=1$.

$$\frac{1}{1+\sin x}+\frac{1}{1+\csc x}$$

$$=\frac{1}{1+\sin x}+\frac{1}{1+\frac{1}{\sin x}} \quad 1$$

$$=\frac{1}{1+\sin x}+\frac{1}{\frac{\sin x+1}{\sin x}} \quad 1\tfrac{1}{2}$$

$$=\frac{1}{1+\sin x}+\frac{\sin x}{1+\sin x} \quad 1\tfrac{1}{2}$$

$$=\frac{1+\sin x}{1+\sin x} \quad 1$$

$$=1 \quad 1$$