

[4 POINTS] Perform the following operations and leave the results in trigonometric form.

(a)
$$\frac{15\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)}{5\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)}$$

$$\frac{15}{5} \operatorname{cis}\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$$

$$= 3 \operatorname{cis}\left(\frac{5\pi}{12}\right)$$

(b)
$$[4(\cos 40^\circ + i \sin 40^\circ)][3(\cos 60^\circ + i \sin 60^\circ)]$$

$$(4 \cdot 3) \operatorname{cis}(40^\circ + 60^\circ)$$

$$= 12 \operatorname{cis} 100^\circ$$

1 POINT EACH

OK TO USE
 $\cos _ + i \sin _$
↓

— $\frac{1}{2}$ POINT IF YOU FORGOT cis (EACH)

— $\frac{1}{2}$ POINT IF YOU FORGOT $^\circ$ IN (b)

[4 POINTS] Convert the polar equation $r = 5 \cos \theta$ to rectangular form and simplify.

$$r = \frac{5x}{r}$$

$$r^2 = 5x$$

$$x^2 + y^2 = 5x$$

OR

$$r^2 = 5r \cos \theta$$

$$x^2 + y^2 = 5x$$

1 POINT EACH

[4 POINTS] Convert the rectangular equation $xy = 4$ to polar form and simplify.

$$\frac{1}{2} (r \cos \theta)(r \sin \theta) = 4$$

$$r^2 \cos \theta \sin \theta = 8$$

$$2r^2 \cos \theta \sin \theta = 8$$

$$r^2 \sin 2\theta = 8$$

$$r^2 = 8 \csc 2\theta$$

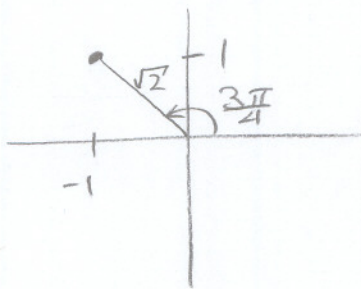
OR

$$r^2 = \frac{4}{\cos \theta \sin \theta}$$

$$r^2 = 4 \sec \theta \csc \theta$$

[3 POINTS] Consider the point with rectangular coordinates $(-1, 1)$.

- (a) Convert the point to polar coordinates.



$$(\sqrt{2}, \frac{3\pi}{4})$$

1 POINT EACH

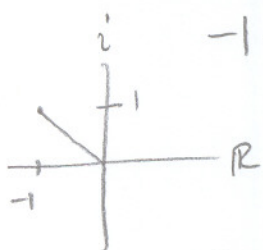
$-\frac{1}{2}$ POINT IF NOT IN
CO-ORDINATE FORM
IE. (r, θ)

- (b) Find a polar representation of the point using the opposite of the r -value you used in part (a).

$$(-\sqrt{2}, -\frac{\pi}{4}) \text{ OR } (-\sqrt{2}, \frac{7\pi}{4})$$

$\frac{1}{2}$ POINT EACH

[5 POINTS] Use DeMoivre's Theorem to find the value of $(-1 + i)^8$. Write the answer in standard form.



$$-1 + i = \sqrt{2} \operatorname{cis} \frac{3\pi}{4} \quad (\text{WORK SAME AS QUESTION ABOVE})$$

$$(-1 + i)^8 = \sqrt{2}^8 \operatorname{cis} (8 \cdot \frac{3\pi}{4})$$

$$= 2^4 \operatorname{cis} 6\pi^{\frac{1}{2}}$$

$$= 16^{\frac{1}{2}} (\cos 6\pi + i \sin 6\pi)$$

$$= 16 (1 + 0i)^{\frac{1}{2}}$$

$$= 16^{\frac{1}{2}}$$

OK TO USE
 $\cos + i \sin$
↓

$-\frac{1}{2}$ POINT IF YOU FORGOT cis