Math 114
Exponentials & Logarithms Review

You should be able to solve the following without a calculator

[1] Find the domains of the following functions.

[a] $f(x) = 4^x$ [b] $f(x) = \log_3 x$

[2] Find the ranges of the following functions.

[a] $f(x) = 5^x$ [b] $f(x) = \log_7 x$

[3] Evaluate the following. Write "UNDEFINED" if the value does not exist.

[a] $\log_3 81$ [b] $\log_6 36$ [c] $\log_5 125$ [d] $\log_7 0$
[e] $\log_8 -8$ [f] $\log_6 1$ [g] $\log_2 64$ [h] $\log 10000$
[i] $\log_4 4^6$ [j] $\log_8 8^{-3}$ [k] $3^{\log_3 7}$ [l] $6^{\log_6 0}$
[m] $5^{\log_5 -10}$ [n] $10^{\log 5}$

[4] Find the exact solutions of the following equations. **Check your answers.**

[a] $3^{2-x} = 81$ [b] $8^{3x-7} = 4^{6-x}$
[c] $1 + 2\log_4(5x+9) = 7$ [d] $\log_3(x^2-7) - \log_3(1-x) = 1$
[e] $\log_2(10x-2) - \log_2(x+1) = 3$ [f] $\log(2x+6) + \log(x-2) = 2$

[5] Write as the logarithm of a single quantity. Simplify your answer.

[a] $\log 8 + \log 5$ [b] $\log 42 - \log 6$
[c] $5\log 2$ [d] $3\log x + 2\log y$
[e] $2\log x - \log y + \log z$ [f] $\log z - 2\log y - \log x$
[g] $2\log y + 3\log z - \log x$ [h] $4\log z + \log x + 3\log y$

[6] Write as the sums and/or differences and/or multiples of logarithms of numbers or single variables.

[a] $\log(7 \times 11)$ [b] $\log\left(\frac{13}{5}\right)$ [c] $\log 3^8$ [d] $\log r^4 s$
[e] $\log\frac{a^5}{b^2}$ [f] $\log\frac{m}{n^2 p^3}$ [g] $\log\frac{x^2}{\sqrt{yz}}$

[7] **MULTIPLE CHOICE**

[a] The graph of $f(x) = 3\log(x+4)$ has an asymptote at
[i] $x = 4$ [ii] $x = -4$ [iii] $y = -4$ [iv] $y = 4$ [v] $y = 3$

[b] The graph of $f(x) = -5 \cdot 2^{x-3}$ has an asymptote at
[i] $x = 3$ [ii] $x = 0$ [iii] $y = -5$ [iv] $y = 3$ [v] $y = 0$

[c] For the logarithm curve $f(x) = \log_2 x$, as the value of $x \to \infty$, the value of $y \to$
[i] ∞ [ii] $-\infty$ [iii] 0 [iv] 1 [v] –1

[d] For the logarithm curve $f(x) = \log_5 x$, as the value of $x \to 0$, the value of $y \to$
[i] ∞ [ii] $-\infty$ [iii] 0 [iv] 1 [v] –1

You may use a non-graphing calculator for the following

[8] Draw the graph of $f(x) = -3 \cdot 2^{-(x-1)}$ by finding and plotting functions values, then sketching the shape of the graph. Show the function values of at least 5 points on your graph. LABEL ALL ASYMPTOTES CLEARLY.

[9] Draw the graph of $f(x) = 2\log_2\left(\frac{x+3}{2}\right)$ by finding and plotting functions values, then sketching the shape of the graph. Show the function values of at least 5 points on your graph. LABEL ALL ASYMPTOTES CLEARLY.

[10] Find the exact solution of the following equations. Also, use your calculator to find a decimal answer, rounded to 4 decimal places. **Check your answers.**

[a] $7^x = 3$ [b] $6^{x-2} = 4^{x+1}$

[11] The number of bacteria in a colony is given by $B(t) = 1.3(2.1)^t$.

[a] How many bacteria were there at time $t = 4$? Round your answer to 1 decimal place.
[b] At what time were there at least 41 bacteria ? Round your answer to 2 decimal places.

[12] [a] Find the intensity (in microns) of an earthquake with a Richter magnitude of 5.6.
[b] Find the Richter magnitude of an earthquake of intensity 56,000,000 microns.

[13] [a] If you deposit \$200 into an account that pays 2.35% interest annually, what is the value of the account 3 years later ?
[b] If you deposit \$200 into an account that pays 2.35% interest annually, when will the value of the account be \$300 ?
[c] How much should you deposit into an account that pays 2.35% interest annually, if you want the value of the account to be \$200 4 years later ?
[b] You deposit \$200 into an account, and 5 years later, the value of the account is \$250. What is the annual interest rate on the account ?

ANSWERS

[1] [a] all real numbers [b] $\{x > 0\}$

[2] [a] $\{y > 0\}$ [b] all real numbers

[3] [a] 4 [b] 2 [c] 3 [d] UNDEFINED
[e] UNDEFINED [f] 0 [g] 6 [h] 4
[i] 6 [j] -3 [k] 7 [l] UNDEFINED
[m] UNDEFINED [n] 5

[4] [a] -2 [b] 3 [c] 11 [d] -5
[e] 5 [f] 7

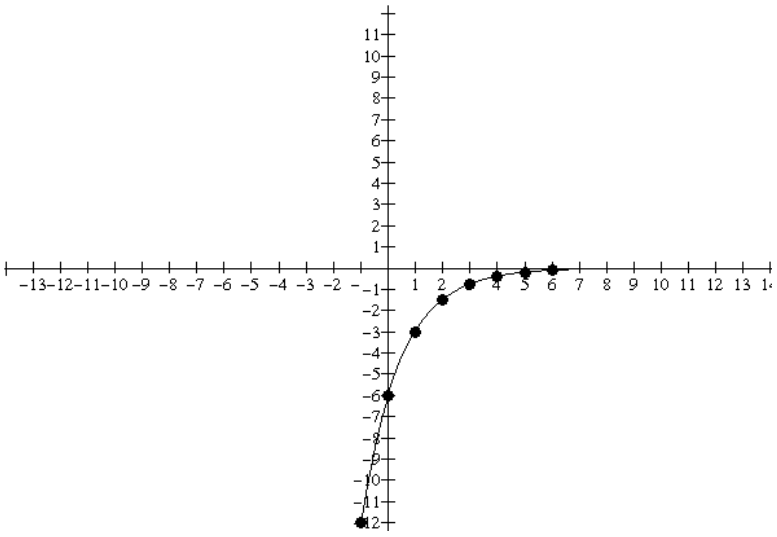
[5] [a] $\log 40$ [b] $\log 7$ [c] $\log 32$ [d] $\log x^3 y^2$
[e] $\log \frac{x^2 z}{y}$ [f] $\log \frac{z}{y^2 x}$ [g] $\log \frac{y^2 z^3}{x}$ [h] $\log z^4 x y^3$

[6] [a] $\log 7 + \log 11$ [b] $\log 13 - \log 5$ [c] $8\log 3$ [d] $4\log r + \log s$
[e] $5\log a - 2\log b$ [f] $\log m - 2\log n - 3\log p$
[g] $2\log x - \frac{1}{2}\log y - \frac{1}{2}\log z$

[7] [a]-[ii] [b]-[v] [c]-[i] [d]-[ii]

[8]

x	–1	0	1	2	3	4	5	6
$f(x)$	–12	–6	–3	–3/2	–3/4	–3/8	–3/16	–3/32

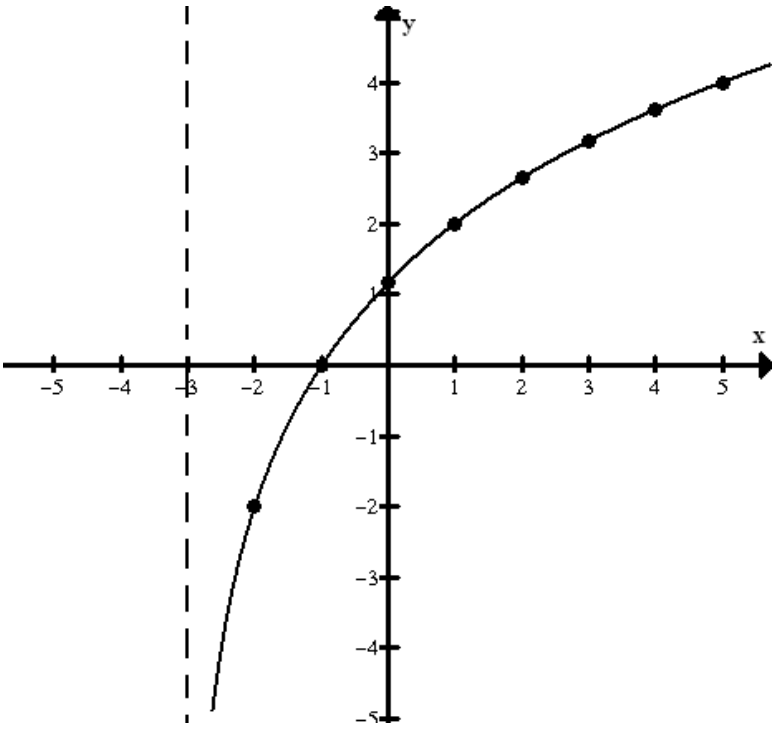


horizontal asymptote at $y = 0$

[9]

x	–2	–1	0	1	2	3	4	5
$f(x)$	–2	0	1.17	2	2.64	3.17	3.61	4

vertical asymptote at $x = -3$



[10] [a] 0.5646 [b] 12.2571

[11] [a] 25.3 bacteria [b] 4.65 units of time

[12] [a] 398107 microns [b] 7.748

[13] [a] \$214.43 [b] 17.46 years later [c] \$182.25 [d] 4.564%