TEXT IN RED IS NOT PART OF THE DEFINITIONS/THEOREMS, AND IS ONLY MEANT FOR CLARIFICATION

DEFINITION OF THE AREA UNDER A CURVE:

If f is a continuous function over $[a, b]$, and $f(x) \geq 0$ on $[a, b]$, then the area under the curve $y = f(x)$ on $[a, b]$ is

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\Delta x$.

[In other words, the area is defined to be the limit of the right handed sum we discussed in class.]

DEFINITION OF A RIEMANN SUM:

Given a regular partition of $[a, b]$ into n equal subintervals, a Riemann sum is any expression of the form

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \dfrac{b-a}{n}$ and $a + (i-1)\Delta x \leq x_i \leq a + i\Delta x$.

The values x_i where the function f is evaluated are called evaluation points.

[In other words, a Riemann sum is a generalization of the right handed sum we have been calculating in class. The Riemann sum is more general, because we can find the values of f at any x-value in each subinterval. And, the area in the first definition above is the limit of the Riemann sum when we use the right sides of each subinterval as the evaluation points.

NOTE: The definition of a Riemann sum does not involve a limit, whereas the definition of area does involve a limit.]

DEFINITION OF A DEFINITE INTEGRAL:

The definite integral of f from a to b is

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \dfrac{b-a}{n}$ and $a + (i-1)\Delta x \leq x_i \leq a + i\Delta x$.

The limit must exist and be the same regardless of how the evaluation points x_i are chosen.

If the limit exists under these circumstances, we say that f is integrable on $[a, b]$.

[In other words, the definite integral (if it exists) is the limit that all Riemann sums of f on $[a, b]$ must approach as the number of subintervals approaches infinity. If the definite integral exists and f is continuous over $[a, b]$, and $f(x) \geq 0$ on $[a, b]$, then the definite integral equals the area under $y = f(x)$ on $[a, b]$.]

INTEGRAL MEAN VALUE THEOREM:

If f is a continuous function over $[a, b]$, then there exist a value c in $[a, b]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

[In other words, a continuous function over an interval always achieves its average value somewhere within that interval.]

FUNDAMENTAL THEOREM OF CALCULUS (PART 1):

If f is a continuous function over $[a, b]$, and F is any anti-derivative of f over $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

[In other words, a definite integral can be evaluated using any anti-derivative of the integrand. This is also referred to as the Evaluation Theorem.]

FUNDAMENTAL THEOREM OF CALCULUS (PART 2):

If f is a continuous function over $[a, b]$, and $F(x) = \int_a^x f(t)dt$ for x in $[a, b]$, then

$$F'(x) = f(x)$$

[In other words, $\frac{d}{dx}\int_a^x f(t)dt = f(x)$.]

FUNDAMENTAL THEOREM OF CALCULUS (PART 3):

If F' is a continuous function over $[a, b]$, then

$$F(b) = F(a) + \int_a^b F'(t)dt$$

[In other words, a differentiable function's value at a certain point can be calculated by adding the function's value at another point plus the definite integral of the function's derivative over the interval between the 2 points (ie. the change in the function between the points). This is also referred to as the Net Change Theorem.

This theorem is not required at this time.]
