Math 1B (7:30am – 8:20am)
Group Quiz 1
Wed Sep 30, 2009

Group Members' Names:
[ Use the name you asked to be called ]

SCORE: ___ / 10 POINTS

**[MULTIPLE CHOICE]** Suppose that the midpoint sum overestimates, and the right endpoint sum underestimates, the area under $f(x)$ on $[a, b]$. Which of the following could describe $f(x)$ ? **SCORE: ___ / 1 POINT**

[a] increasing & concave down
[b] increasing & concave up
[c] decreasing & concave down
[d] decreasing & concave up

LETTER OF CORRECT ANSWER: [C]

Compute the area under $f(x) = 3x^2 - 4x + 3$ over $[-2, 1]$ using the limit of the left endpoint sum. **SCORE: ___ / 9 POINTS**

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(a+(i-1)\Delta x)\Delta x \qquad a=-2 \qquad \Delta x = \frac{1-(-2)}{n} = \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(3\left(-2+\frac{3(i-1)}{n}\right)^2 - 4\left(-2+\frac{3(i-1)}{n}\right)+3\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(3\left(4-\frac{12(i-1)}{n}+\frac{9(i-1)^2}{n^2}\right)+8-\frac{12(i-1)}{n}+3\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(12-\frac{36}{n}i+\frac{36}{n}+\frac{27}{n^2}i^2-\frac{54}{n^2}i+\frac{27}{n^2}+8-\frac{12}{n}i+\frac{12}{n}+3\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \left(\sum_{i=1}^{n}\left(12+\frac{36}{n}+\frac{27}{n^2}+8+\frac{12}{n}+3\right)+\left(-\frac{36}{n}-\frac{54}{n^2}-\frac{12}{n}\right)\sum_{i=1}^{n} i + \frac{27}{n^2}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty} \frac{3}{\cancel{n}} \left(\cancel{n}\left(23+\frac{48}{n}+\frac{27}{n^2}\right)+\left(-\frac{48}{n}-\frac{54}{n^2}\right)\frac{\cancel{n}(n+1)}{2} + \frac{27}{n^2}\frac{\cancel{n}(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} 3\left(23+\frac{48}{n}+\frac{27}{n^2}-\frac{24(n+1)}{n}-\frac{27(n+1)}{n^2}+\frac{9(n+1)(2n+1)}{2n^2}\right)$$

$$= 3(23+0+0-24-0+9)$$

$$= 24$$