

SCORE: ___ / 10 POINTS

[MULTIPLE CHOICE] Suppose that the midpoint sum overestimates, and the right endpoint sum underestimates, SCORE: ___ / 1 POINT
the area under $f(x)$ on $[a, b]$. Which of the following could describe $f(x)$?

[a] increasing &
concave down[b] increasing &
concave up[c] decreasing &
concave down[d] decreasing &
concave up

LETTER OF CORRECT ANSWER: []

Compute the area under $f(x) = 3x^2 - 4x + 3$ over $[-2, 1]$ using the limit of the left endpoint sum.

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LEFT ENDPOINT SUM = RIGHT ENDPOINT SUM
+ "FIRST TERM" - "LAST TERM"

$$\lim_{n \rightarrow \infty} \left[\left(\sum_{i=1}^n f(a+i\Delta x) \Delta x \right) + f(a) \Delta x - f(a+n\Delta x) \Delta x \right]$$

WHERE $\Delta x = \frac{1-(-2)}{n} = \frac{3}{n}$

NOT INSIDE $\sum$ $a = -2$

$$= \lim_{n \rightarrow \infty} \left[\sum_{i=1}^n f\left(-2 + \frac{3i}{n}\right) \frac{3}{n} + f(-2) \frac{3}{n} - f(1) \frac{3}{n} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left[\sum_{i=1}^n \left(3\left(-2 + \frac{3i}{n}\right)^2 - 4\left(-2 + \frac{3i}{n}\right) + 3 \right) + 23 - 2 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left[\sum_{i=1}^n \left(3\left(4 - \frac{12i}{n} + \frac{9i^2}{n^2}\right) + 8 - \frac{12i}{n} + 3 \right) + 21 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left[\sum_{i=1}^n \left(12 - \frac{36i}{n} + \frac{27i^2}{n^2} + 11 - \frac{12i}{n} \right) + 21 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left[\sum_{i=1}^n \left(23 - \frac{48i}{n} + \frac{27i^2}{n^2} \right) + 21 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(23n - \frac{48}{n} \frac{n(n+1)}{2} + \frac{27}{n^2} \frac{n(n+1)(2n+1)}{6} \right) + \frac{63}{n}$$

$$= \lim_{n \rightarrow \infty} 3 \left(23 - \frac{24(n+1)}{n} + \frac{9(n+1)(2n+1)}{2n^2} \right) + \frac{63}{n}$$

$$= 3(23 - 24 + 9) + 0$$

$$= 24$$