
SCORE: ___ / 10 POINTS

[MULTIPLE CHOICE] Suppose that the midpoint sum underestimates, and the left endpoint sum overestimates, the area under $f(x)$ on $[a, b]$. Which of the following could describe $f(x)$? SCORE: ___ / 1 POINT

- [a] decreasing & concave up [b] decreasing & concave down [c] increasing & concave up [d] increasing & concave down

LETTER OF CORRECT ANSWER: [A]

Compute the area under $f(x) = 3x^2 - 6x + 5$ over $[-1, 2]$ using the limit of the left endpoint sum.

SCORE: ___ / 9 POINTS

$$\begin{aligned}
 & \lim_{n \rightarrow \infty} \sum_{i=1}^n f(a + (i-1)\Delta x) \Delta x \quad a = -1 \quad \Delta x = \frac{2 - (-1)}{n} = \frac{3}{n} \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3\left(-1 + \frac{3(i-1)}{n}\right)^2 - 6\left(-1 + \frac{3(i-1)}{n}\right) + 5 \right) \frac{3}{n} \\
 &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(3\left(1 - \frac{6(i-1)}{n} + \frac{9(i-1)^2}{n^2}\right) + 6 - \frac{18(i-1)}{n} + 5 \right) \\
 &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(3 - \frac{18}{n}i + \frac{18}{n} + \frac{27i^2}{n^2} - \frac{54i}{n^2} + \frac{27}{n^2} + 6 - \frac{18}{n}i + \frac{18}{n} + 5 \right) \\
 &= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\sum_{i=1}^n \left(3 + \frac{18}{n} + \frac{27}{n^2} + 6 + \frac{18}{n} + 5 \right) + \left(-\frac{18}{n} - \frac{54}{n^2} - \frac{18}{n} \right) \sum_{i=1}^n i \right. \\
 &\quad \left. + \frac{27}{n^2} \sum_{i=1}^n i^2 \right) \\
 &= \lim_{n \rightarrow \infty} \frac{3}{n} \left(n \left(14 + \frac{36}{n} + \frac{27}{n^2} \right) + \left(-\frac{36}{n} - \frac{54}{n^2} \right) \frac{n(n+1)}{2} \right. \\
 &\quad \left. + \frac{27}{n^2} \frac{n(n+1)(2n+1)}{6} \right) \\
 &= \lim_{n \rightarrow \infty} 3 \left(14 + \frac{36}{n} + \frac{27}{n^2} - \frac{18(n+1)}{n} - \frac{27(n+1)}{n^2} + \frac{9(n+1)(2n+1)}{2n^2} \right) \\
 &= 3(14 + 0 + 0 - 18 - 0 + 9) \\
 &= 15
 \end{aligned}$$