Math 1B (7:30am – 8:20am)
Group Quiz 1
Wed Sep 30, 2009

Group Members' Names:
[Use the name you asked to be called]

SCORE: ___ / 10 POINTS

[MULTIPLE CHOICE] Suppose that the midpoint sum underestimates, and the left endpoint sum overestimates, the area under $f(x)$ on $[a, b]$. Which of the following could describe $f(x)$? **SCORE: ___ / 1 POINT**

[a] decreasing & concave up
[b] decreasing & concave down
[c] increasing & concave up
[d] increasing & concave down

LETTER OF CORRECT ANSWER: [ ]

Compute the area under $f(x) = 3x^2 - 6x + 5$ over $[-1, 2]$ using the limit of the left endpoint sum. **SCORE: ___ / 9 POINTS**

LEFT ENDPOINT SUM = RIGHT ENDPOINT SUM + "FIRST TERM" − "LAST TERM"

$$\lim_{n\to\infty}\left[\left(\sum_{i=1}^{n} f(a+i\Delta x)\Delta x\right) + f(a)\Delta x - f(a+n\Delta x)\Delta x\right]$$

WHERE $\Delta x = \frac{2-(-1)}{n} = \frac{3}{n}$

NOT INSIDE $\sum$

$a = -1$

$$= \lim_{n\to\infty}\left[\sum_{i=1}^{n} f\left(-1+\tfrac{3i}{n}\right)\tfrac{3}{n} + f(-1)\tfrac{3}{n} - f(2)\tfrac{3}{n}\right]$$

$$= \lim_{n\to\infty}\tfrac{3}{n}\left[\sum_{i=1}^{n}\left(3\left(-1+\tfrac{3i}{n}\right)^2 - 6\left(-1+\tfrac{3i}{n}\right)+5\right)+14-5\right]$$

$$= \lim_{n\to\infty}\tfrac{3}{n}\left[\sum_{i=1}^{n}\left(3\left(1-\tfrac{6i}{n}+\tfrac{9i^2}{n^2}\right)+6-\tfrac{18i}{n}+5\right)+9\right]$$

$$= \lim_{n\to\infty}\tfrac{3}{n}\left[\sum_{i=1}^{n}\left(3-\tfrac{18i}{n}+\tfrac{27i^2}{n^2}+11-\tfrac{18i}{n}\right)+9\right]$$

$$= \lim_{n\to\infty}\tfrac{3}{n}\left[\sum_{i=1}^{n}\left(14-\tfrac{36i}{n}+\tfrac{27i^2}{n^2}\right)+9\right]$$

$$= \lim_{n\to\infty}\tfrac{3}{n}\left(14n - \tfrac{36}{n}\cdot\tfrac{n(n+1)}{2} + \tfrac{27}{n^2}\cdot\tfrac{n(n+1)(2n+1)}{6}\right) + \tfrac{27}{n}$$

$$= \lim_{n\to\infty} 3\left(14 - \tfrac{18(n+1)}{n} + \tfrac{9(n+1)(2n+1)}{2n^2}\right) + \tfrac{27}{n}$$

$$= 3(14-18+9)+0$$

$$= 15$$