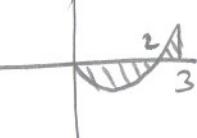

SCORE: ___ / 10 POINTS

Find the area between the graph of $f(x) = x^2 - 2x$ and the x -axis on $[0, 3]$. Simplify your answer.

SCORE: ___ / 3 POINTS



$$\begin{aligned}
 & - \int_0^2 (x^2 - 2x) dx + \int_2^3 (x^2 - 2x) dx \\
 &= - \left(\frac{1}{3}x^3 - x^2 \right) \Big|_0^2 + \left(\frac{1}{3}x^3 - x^2 \right) \Big|_2^3 \\
 &= - \left(\left(\frac{8}{3} - 4 \right) - (0 - 0) \right) + \left((9 - 9) - \left(\frac{8}{3} - 4 \right) \right) \\
 &= \frac{8}{3} \quad \text{OR SIMILAR}
 \end{aligned}$$

Identify the following limit as a Riemann sum, and evaluate the limit. Show all algebraic reasoning.

SCORE: ___ / 3 POINTS

$$\begin{aligned}
 \lim_{n \rightarrow \infty} \frac{5}{n} \left[\frac{1}{\sqrt{4 + \frac{5}{n}}} + \frac{1}{\sqrt{4 + \frac{10}{n}}} + \dots + \frac{1}{\sqrt{4 + \frac{5n}{n}}} \right] &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{\sqrt{4 + \frac{5i}{n}}} \frac{5}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \\
 \Delta x = \frac{5}{n} = \frac{b-a}{n} \Rightarrow b-a=5 &\quad \downarrow \\
 f(x_i) = f(a + i \Delta x) = \frac{1}{\sqrt{4 + \frac{5i}{n}}} & \\
 f(a + \frac{5i}{n}) = \frac{1}{\sqrt{4 + \frac{5i}{n}}} \Rightarrow a=4, b=9 & \\
 f(4 + \frac{5i}{n}) = \frac{1}{\sqrt{4 + \frac{5i}{n}}} \Rightarrow f(x) = \frac{1}{\sqrt{x}} &
 \end{aligned}$$

$$\int_4^9 \frac{1}{\sqrt{x}} dx = 2x^{\frac{1}{2}} \Big|_4^9 = 2\sqrt{9} - 2\sqrt{4} = 2$$

Find the average value of $f(x) = \left(x^{\frac{2}{3}} - 2x^{\frac{1}{3}} \right)^2$ on $[0, 1]$. Simplify your answer.

SCORE: ___ / 4 POINTS

$$\begin{aligned}
 & \frac{1}{1-0} \int_0^1 (x^{\frac{2}{3}} - 2x^{\frac{1}{3}})^2 dx \\
 &= \int_0^1 (x^{\frac{4}{3}} - 4x^{\frac{1}{3}} + 4x^{\frac{2}{3}}) dx \\
 &= \left. \frac{3}{7}x^{\frac{7}{3}} - 2x^{\frac{4}{3}} + \frac{12}{5}x^{\frac{5}{3}} \right|_0^1 \\
 &= \frac{\left(\frac{3}{7} - 2 + \frac{12}{5} \right) - (0 - 0 + 0)}{\frac{1}{2}} = \frac{15 - 70 + 84}{35} = \frac{29}{35} \quad \frac{1}{2}
 \end{aligned}$$