Math 1B (7:30am – 8:20am)
Group Quiz 2
Wed Oct 7, 2009

Group Members' Names:
[Use the name you asked to be called]

SCORE: ___ / 10 POINTS

Find the average value of $f(x) = \left(x^{\frac{2}{3}} - 2x^{\frac{1}{3}}\right)^2$ on $[0, 1]$. Simplify your answer. **SCORE: ___ / 4 POINTS**

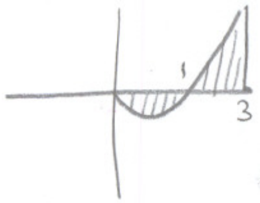
$$\frac{1}{1-0}\int_0^1 (x^{2/3} - 2x^{1/3})^2\,dx \quad 1$$

$$= \int_0^1 (x^{4/3} - 4x + 4x^{2/3})\,dx \quad 1$$

$$= \frac{3}{7}x^{\frac{7}{3}} - 2x^2 + \frac{12}{5}x^{\frac{5}{3}}\Big|_0^1 \quad 1$$

$$= \left(\frac{3}{7} - 2 + \frac{12}{5}\right) - (0 - 0 + 0) \quad \tfrac{1}{2} \quad = \frac{15 - 70 + 84}{35} = \frac{29}{35} \quad \tfrac{1}{2}$$

Find the area between the graph of $f(x) = x^2 - x$ and the x-axis on $[0, 3]$. Simplify your answer. **SCORE: ___ / 3 POINTS**



$$-\int_0^1 (x^2 - x)\,dx + \int_1^3 (x^2 - x)\,dx \quad 1$$

$$= -\left(\frac{1}{3}x^3 - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{3}x^3 - \frac{1}{2}x^2\right)\Big|_1^3 \quad 1$$

$$= -\left(\left(\frac{1}{3} - \frac{1}{2}\right) - (0 - 0)\right) + \left(9 - \frac{9}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right) \quad \tfrac{1}{2}$$

OR SIMILAR

$$= \frac{29}{6} \quad \tfrac{1}{2}$$

Identify the following limit as a Riemann sum, and evaluate the limit. Show all algebraic reasoning. **SCORE: ___ / 3 POINTS**

$$\lim_{n\to\infty} \frac{5}{n}\left[\frac{1}{\sqrt{4 + \frac{5}{n}}} + \frac{1}{\sqrt{4 + \frac{10}{n}}} + \cdots + \frac{1}{3}\right] = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{\sqrt{4 + \frac{5i}{n}}}\,\frac{5}{n} = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$\Delta x = \frac{5}{n} = \frac{b-a}{n} \Longrightarrow b - a = 5$$

$$f(x_i) = f(a + i\Delta x) = \frac{1}{\sqrt{4 + \frac{5i}{n}}}$$

$$f\left(a + \frac{5i}{n}\right) = \frac{1}{\sqrt{4 + \frac{5i}{n}}} \Longrightarrow a = 4,\ b = 9$$

$$f\left(4 + \frac{5i}{n}\right) = \frac{1}{\sqrt{4 + \frac{5i}{n}}} \Longrightarrow f(x) = \frac{1}{\sqrt{x}}$$

$$\int_4^9 \frac{1}{\sqrt{x}}\,dx = 2x^{\frac{1}{2}}\Big|_4^9 = 2\sqrt{9} - 2\sqrt{4} = 2$$