Math 1B
Midterm 1
Review

ADVICE:

You should review everything you've learned (either from lecture, the textbook or the homework). If you only learn a subset of the material, the midterm is likely to seem much longer and harder than it actually is, because you will probably try using the most "obvious" technique (which will take a long time) when a different technique based on a theorem or definition would be much more efficient.

The times below indicate how long you should take for the problem if you really know the chapter well. The [C] indicates problems for which you will definitely be allowed to use your calculator. (All others you should be able to do without a calculator.)

[1] Compute $\sum_{k=1}^{60}(2k^2-3k+1)$ using the rules and shortcuts of summation. **[2 minutes] [C]**

[2] Compute the exact area under the graph of $f(x)=5x+3$ on the interval [3, 7] using the limit of the right-endpoint Riemann sum. **[6 minutes]**

[3] Estimate the area under the graph of f on the interval [2, 8] using 3 subintervals and each of the methods below, given the following function values. **[6 minutes]**

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	7	9	10	13	12	10	7	3	2	2	5

[a] left-endpoint evaluation
[b] midpoint evaluation
[c] right-endpoint evaluation

[4] If f is continuous, increasing and concave down on [a, b], determine whether the following numerical integration methods give answers greater than or less than $\int_a^b f(x)\,dx$. **[4 minutes]**

[a] left hand rule
[b] right hand rule
[c] midpoint rule
[d] trapezoidal rule

[5] Find the area between $y=x^2-4x$ and $y=0$ on the interval [3, 6]. **[5 minutes]**

[6] Find a value c that satisfies the Integral Mean Value Theorem for the function $y=3x^2-2x$ on [1, 3]. **[4 minutes]**

[7] If the average value of the function f on the interval [1, 5] is 3, and the average value on the interval [5, 7] is 2, find the average value on the interval [1, 7]. **[4 minutes]**

[8] Let $f(x)=\int_0^{x^2}\sqrt{16-t^2}\,dt$.

[a] Find $f'(x)$. **[1 minute]**
[b] Find the equation of the tangent line to the graph of $y=f(x)$ at x = 2. **[3 minutes]**

[9] Let $g(t)=\int_{3t}^{t^2}\ln(3x-5)\,dx$. Find $g'(3)$. **[3 minutes]**

[10] Identify and classify all local extrema of $f(x)=\int_5^x\left(\frac{t^2}{4}+t-3\right)dt$. **[3 minutes]**

[11] Identify the following sum as the limit of a Riemann sum and evaluate the limit. **[5 minutes]**

$$\lim_{n\to\infty} \frac{2}{n}\left(e^{\frac{4}{n}} + e^{\frac{8}{n}} + e^{\frac{12}{n}} + \cdots + e^{4} \right)$$

[12] Make the substitution $u = x^{-2}$ for $\int_2^3 \frac{f(x^{-2})}{x^3}\,dx$. **[2 minutes]**

[13] Find

[i] the percentage error and
[ii] bounds on the error

when each of the following rules are used to approximate $\int_1^8 \frac{4}{\sqrt[3]{x}}\,dx$ with $n = 10$. **[6 minutes for each rule] [C]**

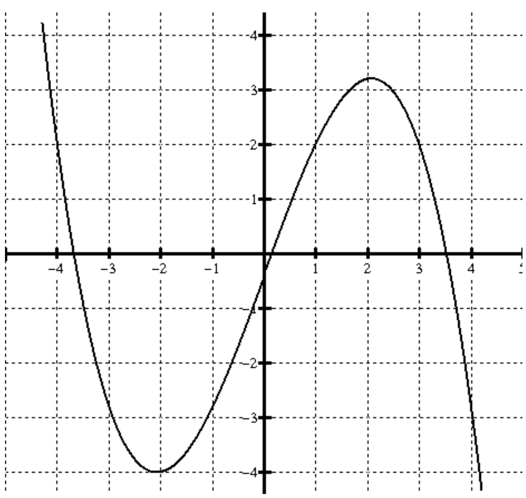
[a] Midpoint Rule
[b] Trapezoidal Rule
[c] Simpson's Rule

[14] Estimate $\int_1^9 f(x)\,dx$ using $n = 4$ and each of the methods below. **[6 minutes]**

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	7	9	10	13	12	10	7	3	2	2	5

[a] Midpoint Rule
[b] Trapezoidal Rule
[c] Simpson's Rule

[15] If $F(x) = \int_1^{x^2} f(t)\,dt$, where $f(t)$ is the function in the graph below, find $F'(2)$. **[2 minutes]**



It would also be helpful to (re-)practice the following integration problems (if you haven't done them already):

Section 4.5	Section 4.6	Review Section
1–20, 21, 24–26	1–40, 41b, 42a, 43b, 44b	1–20, 47–58

In addition, don't forget that you must:

Know the definitions of "area under y = f(x)", "Riemann sum", "definite integral"
Know the text of the Integral Mean Value Theorem and the Fundamental Theorem of Calculus (both parts), and be able to determine if/how they apply to certain situations

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[1] 142190

[2] $\lim_{n\to\infty} \dfrac{112n+40}{n} = 112$

[3] [a] 58 [b] 52 [c] 42

[4] [a] less than [b] greater than [c] greater than [d] less than

[5] $\dfrac{37}{3}$

[6] $\dfrac{1+2\sqrt{7}}{3}$

[7] $\dfrac{8}{3}$

[8] [a] $2x\sqrt{16-x^4}$ [b] $y-4\pi = 0(x-2)$ OR $y = 4\pi$

[9] $3\ln 22$

[10] local maximum at $t=-6$, local minimum at $t=2$

[11] $\dfrac{e^4-1}{2}$

[12] $\displaystyle\int_{\frac{1}{4}}^{\frac{1}{9}} -\frac{f(u)}{2}\,du$ OR $\displaystyle\int_{\frac{1}{9}}^{\frac{1}{4}} \frac{f(u)}{2}\,du$

[13] [a] [i] 0.136% [ii] .2540740741
[b] [i] 0.277% [ii] .5081481481
[c] [i] 0.00162% [ii] .1291072702

[14] [a] 62 [b] 63 [c] $63\frac{1}{3}$

[15] –12