Math 1B (7:30am – 8:20am)
Midterm 1
Tue Oct 13, 2009

SCORE: ___ / 120 POINTS + ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
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NO CALCULATORS ALLOWED

For full credit, you must show the work which leads to all numerical and algebraic answers

Let $F(x) = \int_3^{x^3-5x} f(t)\,dt$, where $f(t)$ is the continuous function in the table below. Find $F'(1)$. **SCORE: ___ / 10 POINTS**

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	7	9	10	13	12	10	7	3	2	2	5
$f'(x)$	1.2	0.5	1.8	-0.4	-1.0	-2.0	-3.0	-1.5	-0.2	2.4	1.2

$$F'(x) = f(x^3-5x)\cdot \frac{d}{dx}(x^3-5x)$$
$$= f(x^3-5x)\cdot(3x^2-5)$$
$$F'(1) = f(-4)\cdot(-2)$$
$$= 9\cdot(-2) = -18$$

Find the average value of $f(x) = \dfrac{(2x+\sqrt{x})^2}{x^2}$ on $[1, 9]$. **SCORE: ___ / 15 POINTS**

$$\frac{1}{9-1}\int_1^9 \frac{(2x+\sqrt{x})^2}{x^2}\,dx$$
$$= \frac{1}{8}\int_1^9 (2+x^{-\frac{1}{2}})^2\,dx$$
$$= \frac{1}{8}\int_1^9 (4+4x^{-\frac{1}{2}}+x^{-1})\,dx$$
$$= \frac{1}{8}\left(4x+8x^{\frac{1}{2}}+\ln|x|\right)\Big|_1^9$$
$$= \frac{1}{8}(36+24+\ln 9-(4+8))$$
$$= \frac{1}{8}(48+\ln 9)$$

Estimate $\int_{-4}^{5} f(x)\,dx$ using Trapezoidal Rule with $n = 3$. **SCORE: ___ / 10 POINTS**

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	7	9	10	13	12	10	7	3	2	1	5
$f'(x)$	1.2	0.5	1.8	-0.4	-1.0	-2.0	-3.0	-1.5	-0.2	2.4	1.2

$$\Delta x = \frac{5--4}{3} = 3$$
$$T_3 = \frac{1}{2}(3)(f(-4)+2f(-1)+2f(2)+f(5))$$
$$= \frac{3}{2}(9+2\cdot 12+2\cdot 3+5)$$
$$= \frac{3}{\cancel{2}}(\overset{22}{\cancel{44}}) = 66$$

Give the complete definition of the definite integral. **SCORE: ___ / 10 POINTS**

THE DEFINITE INTEGRAL OF f OVER $[a,b]$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \quad \text{WHERE } \Delta x = \frac{b-a}{n}$$

$$\text{AND } a+(i-1)\Delta x \le x_i \le a+i\Delta x$$

IF THE LIMIT EXISTS AND IS THE SAME REGARDLESS OF THE CHOICE OF THE x_i.

Compute the exact area under the graph of $f(x) = 6 - 2x$ on the interval $[-1, 2]$ using the limit of the right-endpoint Riemann sum. **SCORE: ___ / 20 POINTS**

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(a+i\Delta x)\Delta x \qquad \Delta x = \frac{2--1}{n} = \frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(6-2\left(-1+\frac{3i}{n}\right)\right)\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(8-\frac{6i}{n}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty}\frac{3}{\cancel{n}}\left(8\cancel{n} - \frac{\overset{3}{\cancel{6}}}{n}\frac{\cancel{n}(n+1)}{\cancel{2}}\right)$$

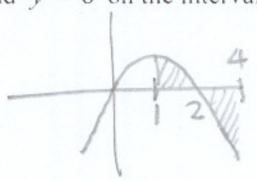
$$= \lim_{n\to\infty} 3\left(8-\frac{3(n+1)}{n}\right)$$

$$= 3(8-3)$$

$$= 15$$

Find the area between $y = 6x - 3x^2$ and $y = 0$ on the interval $[1, 4]$. **SCORE: ___ / 15 POINTS**

$$6x-3x^2=0$$
$$3x(2-x)=0$$
$$x=0,2$$



$$\int_1^2(6x-3x^2)\,dx - \int_2^4(6x-3x^2)\,dx$$

$$= (3x^2-x^3)\Big|_1^2 - (3x^2-x^3)\Big|_2^4$$

$$= (12-8)-(3-1)-((48-64)-(12-8))$$

$$= 4-2-(-16-4)$$

$$= 22$$

Evaluate the following integrals. **SCORE: ___ / 40 POINTS**

[a] $\int \frac{6x^3}{1+x^8}\,dx = \frac{3}{2}\int \frac{1}{1+u^2}\,du = \frac{3}{2}\tan^{-1}u + C = \frac{3}{2}\tan^{-1}x^4 + C$

$u = x^4$

$\frac{du}{dx} = 4x^3$

$dx = \frac{1}{4x^3}\,du$

$\frac{6x^3}{1+x^8}\,dx = \frac{\overset{3}{\cancel{6}}\cancel{x^3}}{1+x^8}\,\frac{1}{\underset{2}{\cancel{4}}\cancel{x^3}}\,du$

$= \frac{3}{2}\,\frac{1}{1+x^8}\,du = \frac{3}{2}\,\frac{1}{1+u^2}\,du$

[b] $\int \frac{4x+1}{\sqrt[3]{3-2x}}\,dx = -\frac{1}{2}\int (7u^{-\frac{1}{3}} - 2u^{\frac{2}{3}})\,du = -\frac{1}{2}(\frac{21}{2}u^{\frac{2}{3}} - \frac{6}{5}u^{\frac{5}{3}}) + C$

$u = 3-2x \implies x = \frac{3-u}{2}$

$= -\frac{21}{4}(3-2x)^{\frac{2}{3}} + \frac{3}{5}(3-2x)^{\frac{5}{3}} + C$

$\frac{du}{dx} = -2$

$dx = -\frac{1}{2}\,du$

$\frac{4x+1}{\sqrt[3]{3-2x}}\,dx = \frac{4x+1}{\sqrt[3]{3-2x}} \cdot -\frac{1}{2}\,du$

$= -\frac{1}{2} \cdot \frac{\overset{2}{\cancel{4}}(\frac{3-u}{\cancel{2}})+1}{u^{\frac{1}{3}}}\,du = -\frac{1}{2}\,\frac{7-2u}{u^{1/3}}\,du = -\frac{1}{2}(7u^{-\frac{1}{3}} - 2u^{\frac{2}{3}})\,du$

[c] $\int_0^1 \frac{6-9x}{3x^2-4x+2}\,dx = \int_2^1 -\frac{3}{2}\cdot\frac{1}{u}\,du = -\frac{3}{2}\ln|u|\Big|_2^1 = -(-\frac{3}{2}\ln 2) = \frac{3}{2}\ln 2$

$u = 3x^2-4x+2 \begin{cases} x=1 \Rightarrow u=1 \\ x=0 \Rightarrow u=2 \end{cases}$

$\frac{du}{dx} = 6x-4$

$dx = \frac{1}{2(3x-2)}\,du$

$\frac{6-9x}{3x^2-4x+2}\,dx = \frac{-3\cancel{(3x-2)}}{3x^2-4x+2} \cdot \frac{1}{2\cancel{(3x-2)}}\,du$

$= -\frac{3}{2} \cdot \frac{1}{u}\,du$

[d] $\int \left(\sin\frac{x}{3} - \sec 2x \tan 2x + \sec^2\frac{3x}{2}\right)dx$

$= -3\cos\frac{x}{3} - \frac{1}{2}\sec 2x + \frac{2}{3}\tan\frac{3x}{2} + C$

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You must turn in the no-calculator portion before ★ you pick up your calculator for this portion ★

Consider the definite integral $\int_0^{\frac{\pi}{4}} 5\sin 2x \, dx$.

SCORE: ___ / 20 POINTS

[a] What is the value of S_{16} ? Round your answer to 4 decimal places.

2.5000

[b] What is the value of M_{20} ? Round your answer to 4 decimal places.

2.5006

[c] Find bounds on the error for the approximation in [b] using the error bounds formula. **Show your work.**
You can leave your simplified answer in terms of π.

$$|E_{M_{20}}| \le \frac{K(b-a)^3}{24n^2}$$

$$\le \frac{20\left(\frac{\pi}{4}-0\right)^3}{24(20)^2}$$

$$= \frac{\frac{\pi^3}{64}}{480}$$

$$= \frac{\pi^3}{30720}$$

$$f = 5\sin 2x$$

$$f' = 10\cos 2x$$

$$f'' = -20\sin 2x$$

$$|f''| = 20\sin 2x \le 20 \cdot 1 = 20 = K$$

BONUS POINTS ON OTHER SIDE