Math 1B (7:30am – 8:20am)
Midterm 2
Thu Nov 5, 2009

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___

SCORE: ___ / 140 POINTS

NO CALCULATORS ALLOWED

For full credit, you must show the work which leads to all numerical and algebraic answers

State both parts of the Fundamental Theorem of Calculus. **SCORE: ___ / -7 POINTS**

YOU WILL SCORE 0 POINTS FOR THIS QUESTION IF IT IS ANSWERED CORRECTLY, AND NEGATIVE 7 POINTS IF IT IS ANSWERED INCORRECTLY.

Give the complete definition of the definite integral. **SCORE: ___ / -7 POINTS**

YOU WILL SCORE 0 POINTS FOR THIS QUESTION IF IT IS ANSWERED CORRECTLY, AND NEGATIVE 7 POINTS IF IT IS ANSWERED INCORRECTLY.

WRITE, BUT DO NOT EVALUATE, AN INTEGRAL FOR THE FOLLOWING PROBLEM

A 20 foot tall water tower is shaped like an upright cone (vertex up). If the base has a radius of 5 feet, write an integral for the amount work done in pumping all the water out through the top of the tower. **SCORE: ___ / 15 POINTS**

USE ρ FOR THE DENSITY OF WATER.

$$\int_0^{20} \rho\pi x\left(\frac{x}{4}\right)^2 dx \text{ OR } \int_0^{20} \rho\pi(20-x)\left(\frac{20-x}{4}\right)^2 dx$$

Find the hydrostatic force on the vertical window of an aquarium if the window is a semicircle of radius 2 foot with the flat edge up and 6 feet below the surface of the water. (The flat edge of the semicircle is 6 feet below the surface of the water.) **SCORE: ___ / 25 POINTS**

USE ρ FOR THE DENSITY OF WATER.

$$\int_0^2 \rho(6+x)(2\sqrt{4-x^2})\,dx$$

$$= 2\rho\left(6\int_0^2 \sqrt{4-x^2}\,dx + \int_0^2 x\sqrt{4-x^2}\,dx\right)$$

$$= 2\rho\left(6(\pi) + \int_4^0 -\frac{1}{2}\sqrt{u}\,du\right)$$ ⬅ SECOND INTEGRAL : LET $u = 4 - x^2$

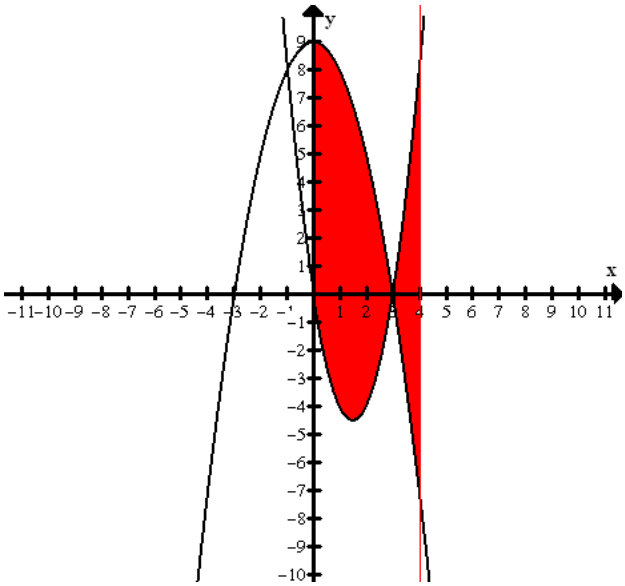
⬆ FIRST INTEGRAL = AREA OF QUARTER CIRCLE OF RADIUS 2

$$= 2\rho\left(6\pi - \frac{1}{3}u^{\frac{3}{2}}\Big|_4^0\right)$$

$$= 2\rho\left(6\pi - -\frac{8}{3}\right)$$

$$= \frac{36\pi + 16}{3}\rho$$

Find the area between the graphs of $y = 2x^2 - 6x$ and $y = 9 - x^2$ on $[0, 4]$. **SCORE: ___ / 20 POINTS**



$$2x^2 - 6x = 9 - x^2$$
$$3x^2 - 6x - 9 = 0$$
$$3(x^2 - 2x - 3) = 0$$
$$3(x-3)(x+1) = 0$$
$$x = -1, 3$$

$$\int_0^3 (9 - x^2 - (2x^2 - 6x))\,dx + \int_3^4 (2x^2 - 6x - (9 - x^2))\,dx$$

$$= \int_0^3 (9 + 6x - 3x^2)\,dx + \int_3^4 (3x^2 - 6x - 9)\,dx$$

$$= (9x + 3x^2 - x^3)\Big|_0^3 + (x^3 - 3x^2 - 9x)\Big|_3^4$$

$$= (27 + 27 - 27) + ((64 - 48 - 36) - (27 - 27 - 27))$$

$$= 27 + (-20 + 27)$$

$$= 34$$

Find the length of the curve $y = \int_0^x \sqrt{(t+5)(t+3)}\ dt$ on $[1, 4]$. **SCORE: ___ / 20 POINTS**

$$\frac{dy}{dx} = \frac{d}{dx}\int_0^x \sqrt{(t+5)(t+3)}\ dt = \sqrt{(x+5)(x+3)}$$

$$\int_1^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$$

$$= \int_1^4 \sqrt{1+(\sqrt{(x+5)(x+3)})^2}\ dx$$

$$= \int_1^4 \sqrt{1+(x+5)(x+3)}\ dx$$

$$= \int_1^4 \sqrt{1+x^2+8x+15}\ dx$$

$$= \int_1^4 \sqrt{x^2+8x+16}\ dx$$

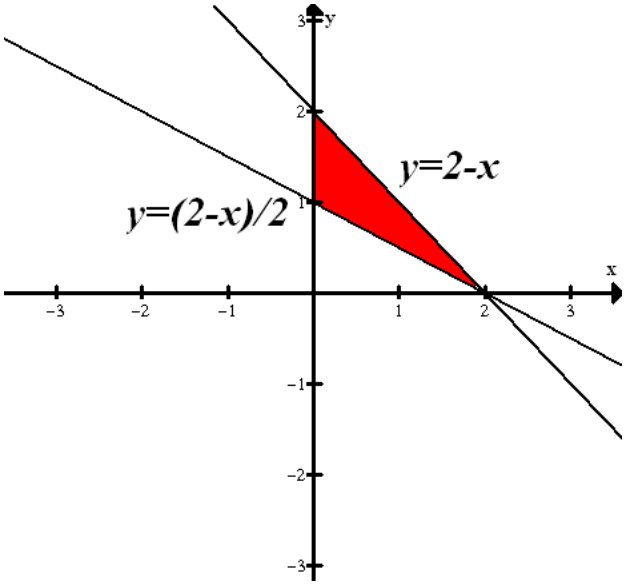
$$= \int_1^4 (x+4)\ dx$$

$$= \left(\frac{x^2}{2}+4x\right)\Big|_1^4$$

$$= (8+16)-(\frac{1}{2}+4)$$

$$= \frac{39}{2}$$

The base of a solid is the area in the xy–plane bounded by $x+y=2$, $x+2y=2$ and $x=0$. Cross sections perpendicular to the *x*-axis are equilateral triangles. Find the volume of the solid. **SCORE: ___ / 20 POINTS**



$$\int_0^2 \frac{\sqrt{3}}{4}\left((2-x)-\frac{2-x}{2}\right)^2\ dx$$

$$= \int_0^2 \frac{\sqrt{3}}{4}\left(\frac{2-x}{2}\right)^2\ dx$$

$$= \frac{\sqrt{3}}{16}\int_0^2 (2-x)^2\ dx$$

$$= \frac{\sqrt{3}}{16}\int_0^2 (4-4x+x^2)\ dx$$

$$= \frac{\sqrt{3}}{16}\left(4x-2x^2+\frac{x^3}{3}\right)\Big|_0^2$$

$$= \frac{\sqrt{3}}{16}\left(8-8+\frac{8}{3}\right)$$

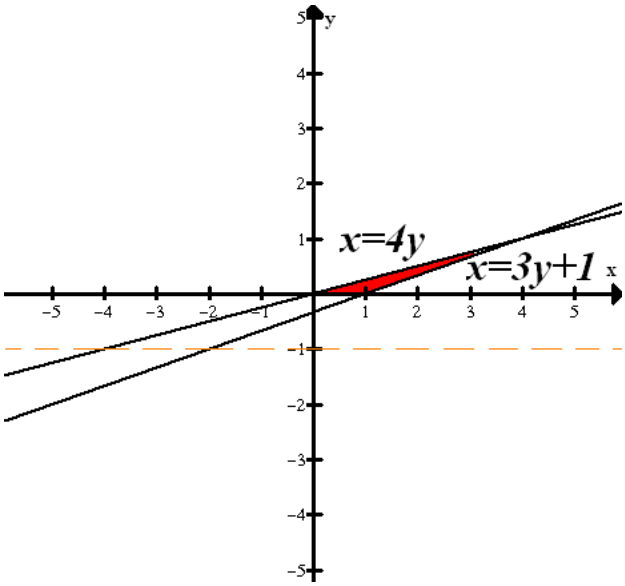
$$= \frac{\sqrt{3}}{6}$$

The area bounded by $y=\frac{x-1}{3}$, $y=\frac{x}{4}$ and $y=0$ is rotated around $y=-1$. Find the volume of the solid. **SCORE: ___ / 20 POINTS**

$x=3y+1 \qquad x=4y$

$3y+1=4y$

$y=1$



$$\int_0^1 2\pi(y--1)(3y+1-4y)\,dy$$

$$=2\pi\int_0^1 (y+1)(1-y)\,dy$$

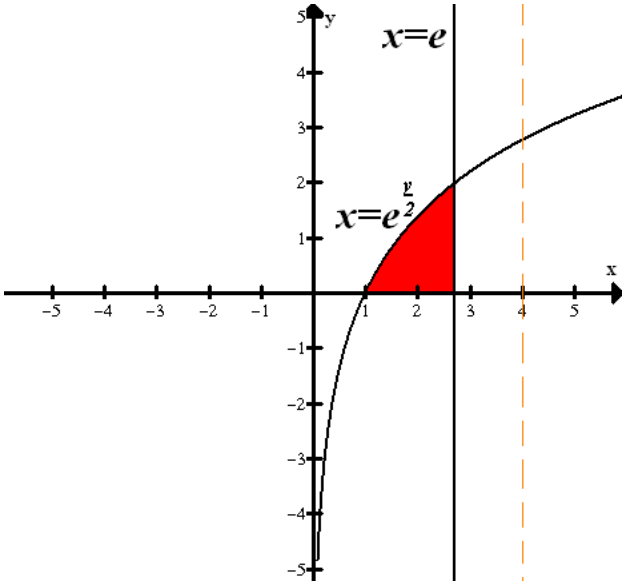
$$=2\pi\int_0^1 (1-y^2)\,dy$$

$$=2\pi\left(y-\frac{y^3}{3}\right)\Big|_0^1$$

$$=2\pi\left(1-\frac{1}{3}\right)$$

$$=\frac{4\pi}{3}$$

The area bounded by $y=2\ln x$, $y=0$ and $x=e$ is rotated around $x=4$. Find the volume of the solid. **SCORE: ___ / 20 POINTS**



$$\int_0^2 \pi\left(\left(4-e^{\frac{y}{2}}\right)^2-(4-e)^2\right)dy$$

$$=\pi\int_0^2\left(16-8e^{\frac{y}{2}}+e^y-(16-8e+e^2)\right)dy$$

$$=\pi\int_0^2\left(-8e^{\frac{y}{2}}+e^y+8e-e^2\right)dy$$

$$=\pi\left(-16e^{\frac{y}{2}}+e^y+8ey-e^2y\right)\Big|_0^2$$

$$=\pi(-16e+e^2+16e-2e^2-(-16+1))$$

$$=\pi(15-e^2)$$