Math 1B (7:30am – 8:20am)
Midterm 2
Thu Nov 5, 2009

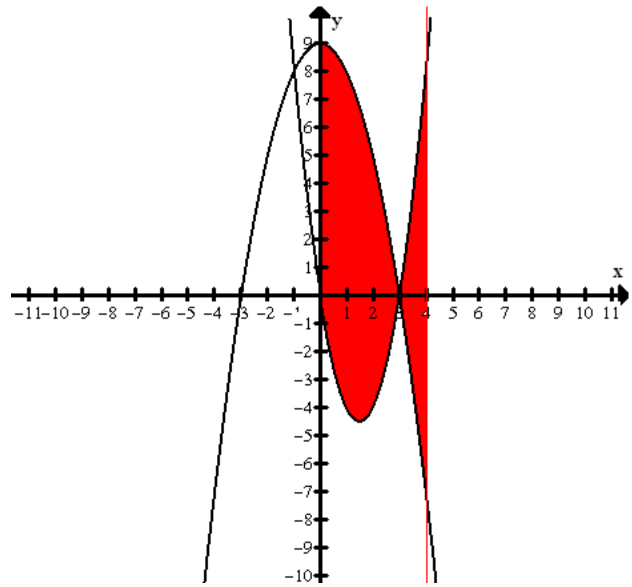
What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___

SCORE: ___ / 140 POINTS

NO CALCULATORS ALLOWED

For full credit, you must show the work which leads to all numerical and algebraic answers

Find the area between the graphs of $y = 9 - x^2$ and $y = 2x^2 - 6x$ on $[0, 4]$. **SCORE: ___ / 20 POINTS**



$$2x^2 - 6x = 9 - x^2$$
$$3x^2 - 6x - 9 = 0$$
$$3(x^2 - 2x - 3) = 0$$
$$3(x - 3)(x + 1) = 0$$
$$x = -1, 3$$

$$\int_0^3 (9 - x^2 - (2x^2 - 6x))\,dx + \int_3^4 (2x^2 - 6x - (9 - x^2))\,dx$$
$$= \int_0^3 (9 + 6x - 3x^2)\,dx + \int_3^4 (3x^2 - 6x - 9)\,dx$$
$$= (9x + 3x^2 - x^3)\Big|_0^3 + (x^3 - 3x^2 - 9x)\Big|_3^4$$
$$= (27 + 27 - 27) + ((64 - 48 - 36) - (27 - 27 - 27))$$
$$= 27 + (-20 + 27)$$
$$= 34$$

Find the length of the curve $y = \int_0^x \sqrt{(t+3)(t+5)}\, dt$ on $[1, 4]$. **SCORE: ___ / 20 POINTS**

$$\frac{dy}{dx} = \frac{d}{dx}\int_0^x \sqrt{(t+5)(t+3)}\, dt = \sqrt{(x+5)(x+3)}$$

$$\int_1^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$
$$= \int_1^4 \sqrt{1 + (\sqrt{(x+5)(x+3)})^2}\, dx$$
$$= \int_1^4 \sqrt{1 + (x+5)(x+3)}\, dx$$
$$= \int_1^4 \sqrt{1 + x^2 + 8x + 15}\, dx$$
$$= \int_1^4 \sqrt{x^2 + 8x + 16}\, dx$$
$$= \int_1^4 (x + 4)\, dx$$

$$= \left(\frac{x^2}{2} + 4x\right)\Bigg|_1^4$$
$$= (8 + 16) - (\frac{1}{2} + 4)$$
$$= \frac{39}{2}$$

The area bounded by $y = 2\ln x$, $y = 0$ and $x = e$ is rotated around $x = 4$. Find the volume of the solid. **SCORE: ___ / 20 POINTS**

$$\int_0^2 \pi\left(\left(4-e^{\frac{y}{2}}\right)^2 - (4-e)^2\right)dy$$

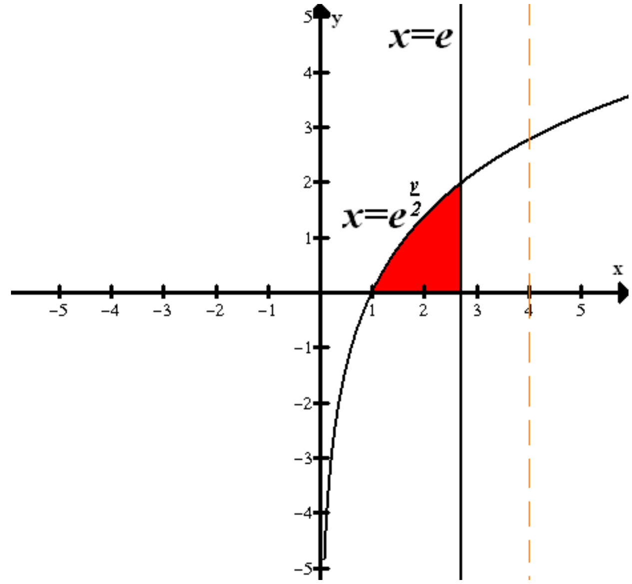
$$= \pi\int_0^2 \left(16 - 8e^{\frac{y}{2}} + e^y - (16 - 8e + e^2)\right)dy$$

$$= \pi\int_0^2 \left(-8e^{\frac{y}{2}} + e^y + 8e - e^2\right)dy$$

$$= \pi\left(-16e^{\frac{y}{2}} + e^y + 8ey - e^2 y\right)\Big|_0^2$$

$$= \pi(-16e + e^2 + 16e - 2e^2 - (-16+1))$$

$$= \pi(15 - e^2)$$



The base of a solid is the area in the xy–plane bounded by $x + 2y = 2$, $x + y = 2$ and $x = 0$. Cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of the solid. **SCORE: ___ / 20 POINTS**

$$\int_0^2 \frac{\sqrt{3}}{4}\left((2-x) - \frac{2-x}{2}\right)^2 dx$$

$$= \int_0^2 \frac{\sqrt{3}}{4}\left(\frac{2-x}{2}\right)^2 dx$$

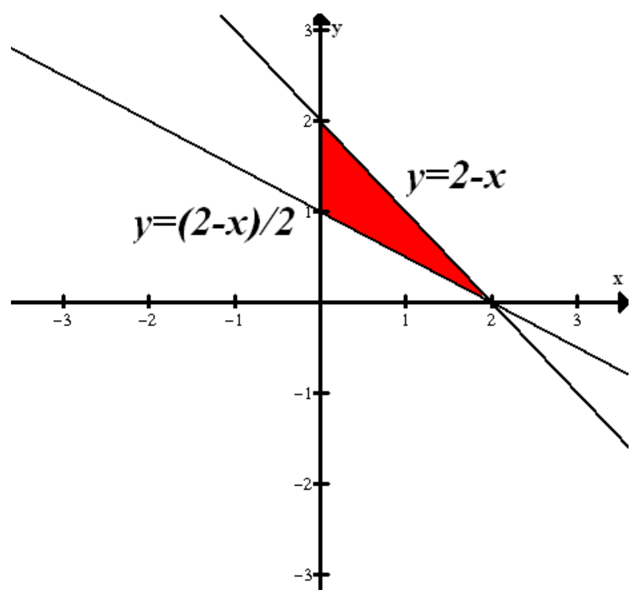
$$= \frac{\sqrt{3}}{16}\int_0^2 (2-x)^2\, dx$$

$$= \frac{\sqrt{3}}{16}\int_0^2 (4 - 4x + x^2)\, dx$$

$$= \frac{\sqrt{3}}{16}\left(4x - 2x^2 + \frac{x^3}{3}\right)\Big|_0^2$$

$$= \frac{\sqrt{3}}{16}\left(8 - 8 + \frac{8}{3}\right)$$

$$= \frac{\sqrt{3}}{6}$$



Find the hydrostatic force on the vertical window of an aquarium if the window is a semicircle of radius 2 foot with the flat edge up and 6 feet below the surface of the water. (The flat edge of the semicircle is 6 feet below the surface of the water.) **SCORE: ___ / 25 POINTS**

USE ρ FOR THE DENSITY OF WATER.

$$\int_0^2 \rho(6+x)(2\sqrt{4-x^2})\,dx$$

$$= 2\rho\left(6\int_0^2 \sqrt{4-x^2}\,dx + \int_0^2 x\sqrt{4-x^2}\,dx\right)$$

$$= 2\rho\left(6(\pi) + \int_4^0 -\frac{1}{2}\sqrt{u}\,du\right)$$ ⬅ **SECOND INTEGRAL : LET $u = 4 - x^2$**

⬆ **FIRST INTEGRAL = AREA OF QUARTER CIRCLE OF RADIUS 2**

$$= 2\rho\left(6\pi - \frac{1}{3}u^{\frac{3}{2}}\Big|_4^0\right)$$

$$= 2\rho\left(6\pi - -\frac{8}{3}\right)$$

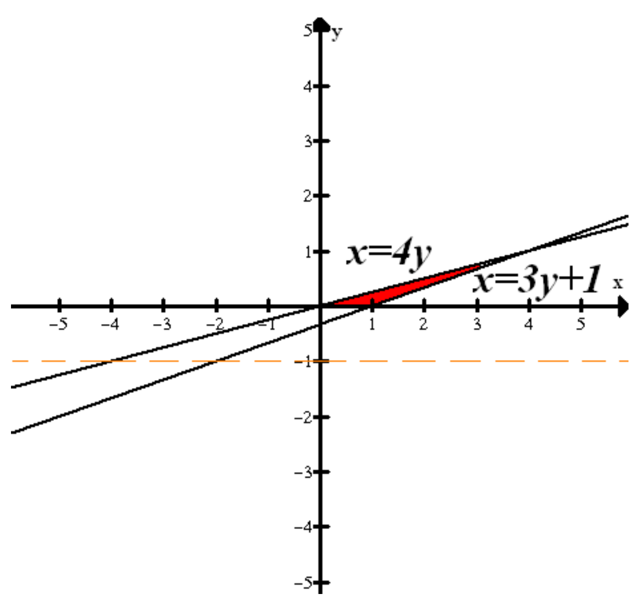
$$= \frac{36\pi + 16}{3}\rho$$

The area bounded by $y = \frac{x}{4}$, $y = \frac{x-1}{3}$ and $y = 0$ is rotated around $y = -1$. Find the volume of the solid. **SCORE: ___ / 20 POINTS**

$x = 3y + 1 \qquad x = 4y$

$3y + 1 = 4y$

$y = 1$



$$\int_0^1 2\pi(y - -1)(3y + 1 - 4y)\,dy$$

$$= 2\pi\int_0^1 (y+1)(1-y)\,dy$$

$$= 2\pi\int_0^1 (1-y^2)\,dy$$

$$= 2\pi\left(y - \frac{y^3}{3}\right)\Big|_0^1$$

$$= 2\pi\left(1 - \frac{1}{3}\right)$$

$$= \frac{4\pi}{3}$$

WRITE, BUT DO NOT EVALUATE, AN INTEGRAL FOR THE FOLLOWING PROBLEM

A 20 foot tall water tower is shaped like an upright cone (vertex up). If the base has a radius of 5 feet, write an integral for the amount work done in pumping all the water out through the top of the tower.

SCORE: ___ / 15 POINTS

USE ρ FOR THE DENSITY OF WATER.

$$\int_0^{20} \rho \pi x \left(\frac{x}{4}\right)^2 dx \text{ OR } \int_0^{20} \rho \pi (20-x)\left(\frac{20-x}{4}\right)^2 dx$$

Give the complete definition of the definite integral.

SCORE: ___ / -7 POINTS

YOU WILL SCORE 0 POINTS FOR THIS QUESTION IF IT IS ANSWERED CORRECTLY, AND NEGATIVE 7 POINTS IF IT IS ANSWERED INCORRECTLY.

State both parts of the Fundamental Theorem of Calculus.

SCORE: ___ / -7 POINTS

YOU WILL SCORE 0 POINTS FOR THIS QUESTION IF IT IS ANSWERED CORRECTLY, AND NEGATIVE 7 POINTS IF IT IS ANSWERED INCORRECTLY.