Math 1B (7:30am – 8:20am)
Midterm 3
Wed Nov 25, 2009

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___

NO CALCULATORS ALLOWED
YOU MUST HAND IN THIS SECTION TO RECEIVE THE TABLE OF INTEGRALS

You may or may not need the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du \qquad \text{if } n \neq 0$$

$$\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u\sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du \qquad \text{if } n \neq 0$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u\tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du \qquad \text{if } n \neq 1$$

Alejandro is expecting an important delivery from FedEqs. FedEqs has guaranteed to make the delivery between 10am and noon, so Alejandro arrives home at 10am to wait for them. Let X be Alejandro's wait time in hours. The probability density function for X is $p(x) = k\sin\frac{\pi x}{4}$. **Assume FedEqs will actually meet their time commitment** 😄 😊 😉.

SCORE: ___ / 20 POINTS

[a] Find the probability Alejandro's delivery will arrive by 11:00am. **Summarize your answer.**

ALEJANDRO MIGHT HAVE TO WAIT 2 HOURS, SO $X \in [0,2]$

$\int_0^2 k\sin\frac{\pi x}{4}\,dx = 1$

$k \cdot -\frac{4}{\pi}\cos\frac{\pi x}{4}\Big|_0^2 = 1$

$-\frac{4k}{\pi}\left(\cos\frac{\pi}{2} - \cos 0\right) = 1$

$\frac{4k}{\pi} = 1$

$k = \frac{\pi}{4}$

$P(X \le 1) = \int_0^1 \frac{\pi}{4}\sin\frac{\pi x}{4}\,dx$

$= -\cos\frac{\pi x}{4}\Big|_0^1$

$= -\cos\frac{\pi}{4} - -\cos 0$

$= 1 - \frac{\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$

THERE IS A $50(2-\sqrt{2})$% CHANCE HIS DELIVERY ARRIVES BY 11AM.

[b] Find Alejandro's expected/mean wait time. **Summarize your answer.**

$\int_0^2 \frac{\pi}{4}x\sin\frac{\pi x}{4}\,dx = -x\cos\frac{\pi x}{4} + \frac{4}{\pi}\sin\frac{\pi x}{4}\Big|_0^2$

$= -2\cos\frac{\pi}{2} - 0 + \frac{4}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right)$

$= \frac{4}{\pi}$

u		dv
$\frac{\pi}{4}x$		$\sin\frac{\pi x}{4}$
	+	
$\frac{\pi}{4}$		$-\frac{4}{\pi}\cos\frac{\pi x}{4}$
	−	
0		$-\frac{16}{\pi^2}\sin\frac{\pi x}{4}$

ALEJANDRO SHOULD EXPECT TO WAIT $\frac{4}{\pi}$ HOURS.

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Evaluate $\int \frac{21-2x^2}{x^2-6x+9}\,dx$.

SCORE: ___ / 15 POINTS

$$= \int \left(-2 + \frac{-12x+39}{(x-3)^2}\right) dx$$

$$= \int \left(-2 + \frac{A}{x-3} + \frac{B}{(x-3)^2}\right) dx$$

$$= \int \left(-2 - \frac{12}{x-3} + \frac{3}{(x-3)^2}\right) dx$$

$$= -2x - 12\ln|x-3| - \frac{3}{x-3} + C$$

$$\begin{array}{r|l} & -2 \\ x^2-6x+9 & -2x^2 \quad +21 \\ & -2x^2+12x-18 \\ \hline & -12x+39 \end{array}$$

$A(x-3) + B = -12x + 39$

$x=3$: $B = 3$

$x=4$: $A + 3 = -9$

$A = -12$

Evaluate $\int x^2 (\ln x)^2\,dx$.

SCORE: ___ / 15 POINTS

u		dv
$(\ln x)^2$	$+$	x^2
$\frac{2\ln x}{x}$		$\frac{1}{3}x^3$
$2\ln x$	$-$	$\frac{1}{3}x^2$
$\frac{2}{x}$		$\frac{1}{9}x^3$
2	$+$	$\frac{1}{9}x^2$
0		$\frac{1}{27}x^3$

$$\frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3(\ln x) + \frac{2}{27}x^3 + C$$

Evaluate $\int (\sin^5 x)\sqrt{\cos x}\, dx$.

SCORE: ___ / 15 POINTS

$u = \cos x$

$du = -\sin x\, dx$

$-\int \sin^4 x \sqrt{\cos x}\,(-\sin x\, dx)$

$= -\int (1-u^2)^2 \sqrt{u}\, du$

$= -\int (1-2u^2+u^4)\, u^{\frac{1}{2}}\, du$

$= -\int (u^{\frac{1}{2}} - 2u^{\frac{5}{2}} + u^{\frac{9}{2}})\, du$

$= -\left(\frac{2}{3}u^{\frac{3}{2}} - \frac{4}{7}u^{\frac{7}{2}} + \frac{2}{11}u^{\frac{11}{2}}\right) + C$

$= -\frac{2}{3}\cos^{\frac{3}{2}} x + \frac{4}{7}\cos^{\frac{7}{2}} x - \frac{2}{11}\cos^{\frac{11}{2}} x + C$

Evaluate $\int \frac{1}{x^3\sqrt{x^2-25}}\, dx$.

SCORE: ___ / 15 POINTS

$x = 5\sec\theta \implies x^2 - 25 = 25\tan^2\theta$

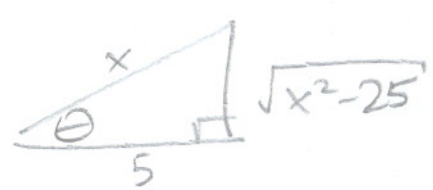
$dx = 5\sec\theta\tan\theta\, d\theta$

$\int \frac{1}{(5\sec\theta)^{\cancel{3}\,2}\, \cancel{5\tan\theta}}\, \cancel{5\sec\theta}\,\cancel{\tan\theta}\, d\theta$

$= \frac{1}{125}\int \frac{1}{\sec^2\theta}\, d\theta$

$= \frac{1}{125}\int \cos^2\theta\, d\theta$

$\sec\theta = \frac{x}{5}$



$= \frac{1}{125}\left(\frac{1}{2}\cos\theta\sin\theta + \frac{1}{2}\theta\right) + C$

$= \frac{1}{250}\left(\frac{5}{x}\,\frac{\sqrt{x^2-25}}{x} + \sec^{-1}\frac{x}{5}\right) + C$

$= \frac{\sqrt{x^2-25}}{50x^2} + \frac{1}{250}\sec^{-1}\frac{x}{5} + C$

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Evaluate $\int e^{-3x} \sin 2x \, dx$. $= -\frac{1}{3}e^{-3x}\sin 2x - \frac{2}{9}e^{-3x}\cos 2x$

SCORE: ___ / 15 POINTS

$$-\frac{4}{9}\int e^{-3x}\sin 2x \, dx$$

u		dv
$\sin 2x$	$+$	e^{-3x}
$2\cos 2x$	$-$	$-\frac{1}{3}e^{-3x}$
$-4\sin 2x$	$+$	$\frac{1}{9}e^{-3x}$

$$\frac{13}{9}\int e^{-3x}\sin 2x \, dx = -\frac{1}{3}e^{-3x}\sin 2x - \frac{2}{9}e^{-3x}\cos 2x$$

$$\int e^{-3x}\sin 2x \, dx = -\frac{3}{13}e^{-3x}\sin 2x - \frac{2}{13}e^{-3x}\cos 2x + C$$

Evaluate $\int \frac{5x-2}{x^2+4x+5} dx$.

SCORE: ___ / 15 POINTS

$$= \int \frac{A(2x+4)+B}{x^2+4x+5} dx$$

$5x-2 = A(2x+4)+B$

$x=-2: \ -12 = B$

$x=0: \ -2 = 4A-12$

$10 = 4A$

$A = \frac{5}{2}$

$$= \int \frac{\frac{5}{2}(2x+4)-12}{x^2+4x+5} dx$$

$$= \frac{5}{2}\int \frac{2x+4}{x^2+4x+5} dx - 12\int \frac{1}{(x+2)^2+1} dx$$

$$= \frac{5}{2}\ln(x^2+4x+5) - 12\tan^{-1}(x+2) + C$$

SUBMIT THIS SECTION TO RECEIVE THE TABLE OF INTEGRALS

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YOU MAY USE THE TABLE OF INTEGRALS ON THIS SECTION
NO CALCULATORS ALLOWED
SIMPLIFY YOUR ANSWERS

Evaluate $\int x^5\sqrt{9-x^4}\,dx$. **SCORE: ___ / 15 POINTS**

If you use the table of integrals, state the number of the rule you are using. YOU DO NOT NEED TO WRITE THE ENTIRE RULE.

$u = x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$\int x^4\sqrt{9-x^4}\,x\,dx$

$= \frac{1}{2}\int u^2\sqrt{9-u^2}\,du$ 30 $a=3$

$= \frac{1}{2}\left(\frac{1}{8}u(2u^2-9)\sqrt{9-u^2} + \frac{1}{8}\cdot 81\sin^{-1}\frac{u}{3}\right)+C$

$= \frac{1}{16}\left(x^2(2x^4-9)\sqrt{9-x^4} + 81\sin^{-1}\frac{x^2}{3}\right)+C$

Evaluate $\int \frac{\sin x\cos^2 x}{\sqrt{\cos^2 x+4}}\,dx$. **SCORE: ___ / 15 POINTS**

If you use the table of integrals, state the number of the rule you are using. YOU DO NOT NEED TO WRITE THE ENTIRE RULE.

$u = \cos x$
$du = -\sin x\,dx$

$-\int \frac{\cos^2 x}{\sqrt{\cos^2 x+4}}(-\sin x)\,dx$

$= -\int \frac{u^2}{\sqrt{u^2+4}}\,du$ 26 $a=2$

$= -\left(\frac{1}{2}u\sqrt{4+u^2} - \frac{1}{2}4\ln\left|u+\sqrt{4+u^2}\right|\right)+C$

$= -\frac{1}{2}\cos x\sqrt{4+\cos^2 x} + 2\ln\left(\cos x + \sqrt{4+\cos^2 x}\right)+C$

BONUS POINTS ON OTHER SIDE