Math 1B
Midterm 3 Probability Review

READ THE COMMENT ON MY WEBSITE ABOUT HOW TO STUDY CHAPTER 6

A computer battery manufacturer guarantees that their batteries last up to one year (ie. their batteries are guaranteed to be dead after one year). Let X be the lifespan (in years) of a randomly selected battery. The probability density function for X is

[1] ce^{-x} [easier integrals]

[2] $\dfrac{k}{4x^2+4x+4}$ [harder integrals involving fractions and radicals]

[a] Find the probability that a randomly selected battery lasts more than 6 months. **Summarize your answer.**
[b] Find the median lifespan of a battery. **Summarize your answer.**
[c] Find the mean lifespan of a battery. **Summarize your answer.**

HINT: For [2abc], you will repeatedly encounter constant multiples of the same integral. Reuse old work to reduce your total work. And remember, factoring is your friend.

ANSWERS:

[1a] The probability that a randomly selected battery lasts more than 6 months is $\dfrac{1}{1+\sqrt{e}} \approx 0.378 \approx 37.8\%$. In other words, 37.8% of all batteries from this manufacturer last more than 6 months.

[1b] The median lifespan of a randomly selected battery is $\ln\left(\dfrac{2e}{e+1}\right) \approx 0.380$ years, or $12\ln\left(\dfrac{2e}{e+1}\right) \approx 4.559$ months. In other words, half of all batteries from this manufacturer last more than 4.559 months, and half last less than 4.559 months.

[1c] The mean lifespan of a randomly selected battery is $\dfrac{e-2}{e-1} \approx 0.418$ years, or $12\left(\dfrac{e-2}{e+1}\right) \approx 5.016$ months. In other words, the average of the lifespans of all batteries from this manufacturer is 5.016 months.

[2a] The probability that a randomly selected battery lasts more than 6 months is $2-\dfrac{6}{\pi}\tan^{-1}\dfrac{2}{\sqrt{3}} \approx 0.363 \approx 36.3\%$. In other words, 36.3% of all batteries from this manufacturer last more than 6 months.

[2b] The median lifespan of a randomly selected battery is $\dfrac{\sqrt{3}-1}{2} \approx 0.366$ years, or $6(\sqrt{3}-1) \approx 4.392$ months. In other words, half of all batteries from this manufacturer last more than 4.392 months, and half last less than 4.392 months.

[2c] The mean lifespan of a randomly selected battery is $\dfrac{3\sqrt{3}\ln 3-\pi}{2\pi} \approx 0.409$ years, or $\dfrac{6(3\sqrt{3}\ln 3-\pi)}{\pi} \approx 4.903$ months. In other words, the average of the lifespans of all batteries from this manufacturer is 4.903 months.