Math 1B (7:30am – 8:20am)
Quiz 3
Fri Oct 9, 2009

SCORE: ___ / 20 POINTS

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your social security number ? __ __
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

To receive full credit, it should be clear how you arrived at all numerical and algebraic answers

[MULTIPLE CHOICE] Which of the following statements does the Fundamental Theorem of Calculus guarantee is true ? **(In other words, which statement satisfies all the "if" conditions of the theorem, and makes a valid conclusion based on the "then" part of the theorem ?)** **SCORE: ___ / 2 POINTS**

[A] $\int_{-4}^{4} \frac{1}{x}\, dx = 0$

[B] If $F(x) = \int_{1}^{x} \frac{1}{t}\, dt$, then $F'(4) = \frac{1}{4}$

[C] The average value of $f(x) = \frac{1}{x}$ on $[1, 4]$ is $\frac{1}{3}\ln 4$

[D] NONE OF THE ABOVE

LETTER OF CORRECT ANSWER: [B]

Let $F(x) = \int_{1}^{e^{2x}} \sqrt{1+t^2}\, dt$. **SCORE: ___ / 3 POINTS**

[a] Find $F'(x)$.

$$F'(x) = \frac{d}{d(e^{2x})}\int_{1}^{e^{2x}} \sqrt{1+t^2}\, dt \cdot \frac{d(e^{2x})}{dx}$$

$$= \sqrt{1+(e^{2x})^2} \cdot 2e^{2x} = \underbrace{2e^{2x}}_{\frac{1}{2}} \underbrace{\sqrt{1+e^{4x}}}_{1}$$

[b] Find the equation of the tangent line to $y = F(x)$ at $x = 0$.

$$F'(0) = 2e^{2(0)}\sqrt{1+e^{4(0)}} = 2(1)\sqrt{1+1} = \underline{2\sqrt{2}}\ \tfrac{1}{2}$$

$$F(0) = \int_{1}^{e^{2(0)}} \sqrt{1+t^2}\, dt = \int_{1}^{1} \sqrt{1+t^2}\, dt = \underline{0}\ \tfrac{1}{2}$$

$$y - 0 = 2\sqrt{2}(x-0) \Longrightarrow \underline{y = 2\sqrt{2}x}\ \tfrac{1}{2}$$

Find $\int \frac{2x+5}{x-3}\, dx$. **SCORE: ___ / 3 POINTS**

$\underline{u = x-3}$ $\frac{1}{2}$ $\longrightarrow x = u+3$

$\underline{du = dx}$ $\frac{1}{2}$

$$\int \frac{2x+5}{x-3}\, dx = \int \underbrace{\frac{2(u+3)+5}{u}}_{\frac{1}{2}}\, du$$

$$= \int \underbrace{\frac{2u+11}{u}}_{\frac{1}{2}}\, du$$

$$= \int \left(2 + \frac{11}{u}\right) du$$

$$= \underbrace{2u + 11\ln|u|}_{\frac{1}{2}} + C$$

$$= \underbrace{2(x-3) + 11\ln|x-3| + C}_{\frac{1}{2}}$$

$$= \underbrace{2x}\ + 11\ln|x-3| + \underbrace{C}$$

+ ½ BONUS IF YOU KNEW THE "−6" COULD BE PART OF "+C"

− ½ POINT IF YOU FORGOT "+C" [NO POINTS IF YOU REMEMBERED "+C"]

CONTINUED ON OTHER SIDE ➔

Find $\int_1^e \frac{\sqrt[3]{\ln x}}{x}\,dx$. **SCORE: ___ / 3 POINTS**

$u = \ln x$, $x = e \Rightarrow u = 1$, $x = 1 \Rightarrow u = 0$

$du = \frac{1}{x}\,dx$

$$\int_1^e \frac{\sqrt[3]{\ln x}}{x}\,dx = \int_0^1 \sqrt[3]{u}\,du = \int_0^1 u^{\frac{1}{3}}\,du = \frac{3}{4}u^{\frac{4}{3}}\Big|_0^1 = \frac{3}{4}$$

$\frac{1}{2}$ POINT EACH

$\frac{3}{4}(\ln x)^{\frac{4}{3}}\Big|_1^e = \frac{3}{4}(\ln e)^{\frac{4}{3}} - \frac{3}{4}(\ln 1)^{\frac{4}{3}} = \frac{3}{4}\cdot 1^{\frac{4}{3}} - \frac{3}{4}\cdot 0^{\frac{4}{3}} = \frac{3}{4}$ IS OK TOO

Find $\int \frac{x^2}{\sqrt{1-x^6}}\,dx$. **SCORE: ___ / 3 POINTS**

$u = x^3$, $\frac{1}{2}$

$du = 3x^2\,dx$, $\frac{1}{2}$

$\frac{1}{3}\,du = x^2\,dx$

$$\int \frac{x^2}{\sqrt{1-x^6}}\,dx = \underbrace{\frac{1}{3}}_{\frac{1}{2}}\int \underbrace{\frac{1}{\sqrt{1-u^2}}\,du}_{\frac{1}{2}} = \underbrace{\frac{1}{3}\sin^{-1} u}_{\frac{1}{2}} + C = \underbrace{\frac{1}{3}\sin^{-1} x^3}_{\frac{1}{2}} + C$$

$-\frac{1}{2}$ POINT IF YOU FORGOT "+ C"

Make the substitution $u = x^{-2}$ for $\int_2^3 \frac{f(x^{-2})}{x^3}\,dx$. Your final answer should be an equivalent integral in u. **SCORE: ___ / 2 POINTS**

$u = x^{-2}$, $x = 3 \Rightarrow u = \frac{1}{9}$, $x = 2 \Rightarrow u = \frac{1}{4}$

$du = -2x^{-3}\,dx$

$dx = -\frac{1}{2}x^3\,du$

$$\frac{f(x^{-2})}{x^3}\,dx = \frac{f(x^{-2})}{\cancel{x^3}}\cdot -\frac{1}{2}\cancel{x^3}\,du = -\frac{1}{2}f(x^{-2})\,du = -\frac{1}{2}f(u)\,du$$

$$-\frac{1}{2}\int_{\frac{1}{4}}^{\frac{1}{9}} f(u)\,du$$

$\frac{1}{2}$ POINT EACH

State both parts of the Fundamental Theorem of Calculus. **SCORE: ___ / 4 POINTS**

IF f IS CONT. ON $[a,b]$ AND F IS ANY ANTI-DERIVATIVE OF f, THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

IF f IS CONT. ON $[a,b]$ AND $F(x) = \int_a^x f(t)\,dt$ THEN $F'(x) = f(x)$