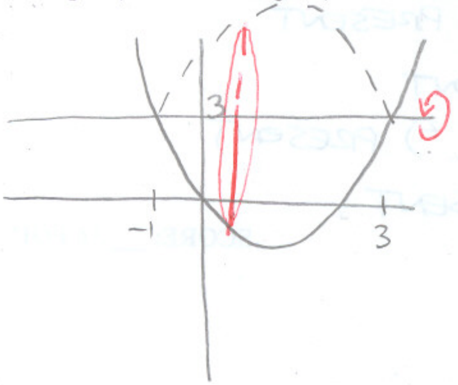
Math 1B (7:30am – 8:20am)
Quiz 4
Fri Oct 23, 2009

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

The region bounded by $y = x^2 - 2x$ and $y = 3$ is revolved around $y = 3$. **SCORE: ___ / 3 POINTS**
Write an integral (or sum of integrals) for the volume. **DO NOT EVALUATE THE INTEGRAL.**

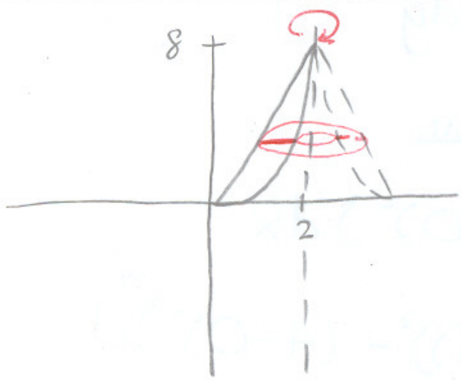


$$\underbrace{\int}_{\frac14}{}_{\underbrace{-1}_{\frac14}}^{\overset{\frac14}{3}} \underbrace{\pi}_{\frac14}\underbrace{(3-(x^2-2x))}_{1\frac12}^{\overset{\frac14}{2}}\,\underbrace{dx}_{\frac14}$$

$$= \int_{-1}^{3} \pi\,(3+2x-x^2)^2\,dx$$

$x^2-2x=3$
$x^2-2x-3=0$
$(x-3)(x+1)=0$
$x=3,-1$

The region in the 1[st] quadrant bounded by $y = x^3$ and $y = 4x$ is revolved around $x = 2$. **SCORE: ___ / 3 POINTS**
Write an integral (or sum of integrals) for the volume **USING EITHER THE DISC OR WASHER METHOD**.
DO NOT EVALUATE THE INTEGRAL.



$\frac14$ IF BOTH PRESENT

$$\underbrace{\int}_{\frac14}{}_{\underbrace{0}_{\frac14}}^{\overset{\frac14}{8}} \underbrace{\pi}_{\frac14}\left(\underbrace{\left(2-\frac{y}{4}\right)}_{\frac34}^{2}-\underbrace{\left(2-\sqrt[3]{y}\right)}_{\frac34}^{2}\right)\underbrace{dy}_{\frac14}$$

$x^3=4x$
$x^3-4x=0$
$x(x+2)(x-2)=0$
$x=0,\pm2$

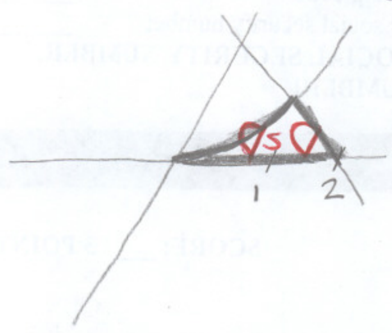
State both parts of the Fundamental Theorem of Calculus. **NO PARTIAL CREDIT.** **SCORE: ___ / 2 POINTS**

IF f IS CONT. ON $[a,b]$ AND F IS ANY ANTI-DERIVATIVE OF f
THEN $\int_a^b f(x)\,dx = F(b)-F(a)$

IF f IS CONT. ON $[a,b]$ AND $F(x)=\int_a^x f(t)\,dt$
THEN $F'(x)=f(x)$

CONTINUED ON OTHER SIDE ➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔

The base of a solid is the area in the xy-plane bounded by $y = x^2$ and $y = 2 - x$. Cross sections perpendicular to the x-axis are semicircles. Write an integral (or sum of integrals) for the volume. **DO NOT EVALUATE THE INTEGRAL.** **SCORE: ___ / 4 POINTS**



$$\int_0^1 \frac{1}{8}\pi\left(x^2-0\right)^2 dx + \int_1^2 \frac{1}{8}\pi\left((2-x)-0\right)^2 dx$$

$$= \int_0^1 \frac{1}{8}\pi (x^2)^2\, dx + \int_1^2 \frac{1}{8}\pi (2-x)^2\, dx$$

$x^2 = 2 - x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

$+\frac{1}{4}$ IF BOTH $\int$ SYMBOLS PRESENT

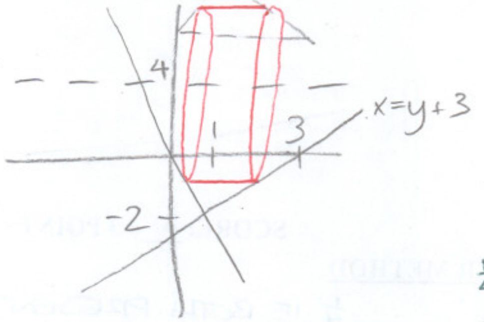
$+\frac{1}{4}$ IF BOTH dx PRESENT

$+\frac{1}{4}$ IF BOTH SQUARES (^{2}) PRESENT

$+\frac{1}{4}$ IF BOTH $\frac{1}{8}\pi$ PRESENT

The region defined by $y \ge -2x$, $y \ge x - 3$ and $y \le 0$ is revolved around $y = 4$. Write an integral (or sum of integrals) for the volume **SCORE: ___ / 6 POINTS**

[a] **USING THE SHELL METHOD**. **DO NOT EVALUATE THE INTEGRAL.**



$$\int_{-2}^{0} 2\pi (4-y)\left(y+3 - -\tfrac{1}{2}y\right) dy$$

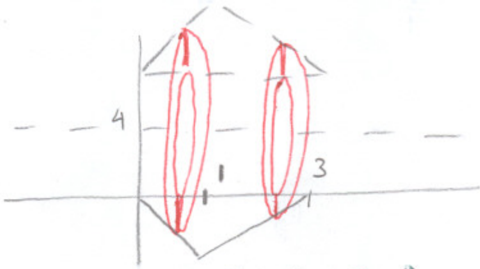
$$= \int_{-2}^{0} 2\pi (4-y)\left(\tfrac{3}{2}y+3\right) dy$$

$-2x = x - 3$

$-3x = -3$

$x = 1$

[b] **USING THE DISC OR WASHER METHOD**. **DO NOT EVALUATE THE INTEGRAL.**



$$\int_0^1 \pi\left((4 - -2x)^2 - (4-0)^2\right) dx$$

$$+ \int_1^3 \pi\left((4-(x-3))^2 - (4-0)^2\right) dx$$

$$= \int_0^1 \pi\left((4+2x)^2 - 16\right) dx + \int_1^3 \pi\left((7-x)^2 - 16\right) dx$$

$+\frac{1}{4}$ IF BOTH $\int$ SYMBOLS PRESENT

$+\frac{1}{4}$ IF BOTH dx PRESENT

$+\frac{1}{4}$ IF BOTH π PRESENT

Give the complete definition of the definite integral. **NO PARTIAL CREDIT.** **SCORE: ___ / 2 POINTS**

THE DEFINITE INTEGRAL OF f OVER $[a, b]$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{WHERE } \Delta x = \frac{b-a}{n}$$

AND $a + (i-1)\Delta x \le x_i \le a + i\Delta x$

IF THE LIMIT EXISTS AND IS THE SAME REGARDLESS OF THE CHOICE OF THE x_i.