Math 1B (7:30am – 8:20am)
Quiz 5
Fri Oct 30, 2009

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___

SCORE: ___ / 20 POINTS

NO CALCULATORS ALLOWED

State the Integral Mean Value Theorem. **NO PARTIAL CREDIT.** **SCORE: ___ / 2 POINTS**

SEE DEFINITIONS AND THEOREMS HANDOUT FOR CORRECT ANSWER

Find the length of the curve $y = x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}$ on $[1, 4]$. **SCORE: ___ / 4 POINTS**

$\int_1^4 \sqrt{1 + \left(\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}}\right)^2}\,dx$ ← **1 PT FOR INTEGRAL**

$= \int_1^4 \sqrt{1 + \left(\frac{1}{4}x^{-1} - \frac{1}{2} + \frac{1}{4}x\right)}\,dx$ ← **½ PT FOR EXPANDING**

$= \int_1^4 \sqrt{\frac{1}{4}x^{-1} + \frac{1}{2} + \frac{1}{4}x}\,dx$ ← **¼ PT FOR SIMPLIFYING**

$= \int_1^4 \left(\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}}\right)dx$ ← **½ PT FOR SIMPLIFYING**

$= \left(x^{\frac{1}{2}} + \frac{1}{3}x^{\frac{3}{2}}\right)\Big|_1^4$ ← **1 PT FOR ANTIDERIVATIVE**

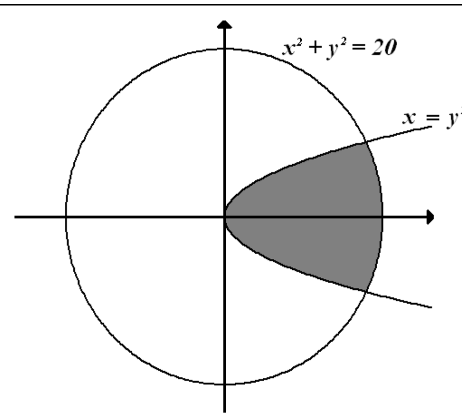
$= \left(2 + \frac{8}{3}\right) - \left(1 + \frac{1}{3}\right)$ ← **¼ PT FOR SUBSTITUTING**

$= 1 + \frac{7}{3}$

$= \frac{10}{3}$ ← **½ PT FOR ANSWER**

The shaded area bounded by $x^2 + y^2 = 20$ and $x = y^2$ is revolved around $x = 0$.
Find the volume of the resulting solid. **SCORE: ___ / 4 POINTS**

$x^2 + x = 20$
$x^2 + x - 20 = 0$
$(x+5)(x-4) = 0$
$x = -5, 4$
$y = \pm 2$ ← **½ PT FOR FINDING LIMITS OF INTEGRATION**



BY WASHER METHOD (SEE OTHER KEY FOR SHELL METHOD)

$\int_{-2}^{2} \pi[(\sqrt{20 - y^2})^2 - (y^2)^2]\,dy$ ← **1½ PT FOR INTEGRAL**

$= \pi\int_{-2}^{2} (20 - y^2 - y^4)\,dy$

$= \pi\left(20y - \frac{y^3}{3} - \frac{y^5}{5}\right)\Big|_{-2}^{2}$ ← **1 PT FOR ANTIDERIVATIVE**

$= \pi\left(20(2 - -2) - \frac{(8 - -8)}{3} - \frac{(32 - -32)}{5}\right)$ ← **½ PT FOR SUBSTITUTING**

$= \pi\left(80 - \frac{16}{3} - \frac{64}{5}\right)$

$= \frac{928}{15}\pi$ ← **½ PT FOR ANSWER**

CONTINUED ON OTHER SIDE ➔

The area under $y = x^2 + 1$ on $[-1, 2]$ is revolved around $x = 2$. Find the volume of the resulting solid. **SCORE: ___ / 4 POINTS**

$\int_{-1}^{2} 2\pi(2-x)(x^2+1)\,dx$ ← **1½ PT FOR INTEGRAL**

$= 2\pi \int_{-1}^{2} (2 - x + 2x^2 - x^3)\,dx$ ← **½ PT FOR EXPANDING**

$= 2\pi\left(2x - \frac{x^2}{2} + \frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_{-1}^{2}$ ← **1 PT FOR ANTIDERIVATIVE**

$= 2\pi\left(2(2--1) - \frac{(4-1)}{2} + \frac{2(8--1)}{3} - \frac{(16-1)}{4}\right)$

$= 2\pi\left(6 - \frac{3}{2} + 6 - \frac{15}{4}\right)$ ↑ **½ PT FOR SUBSTITUTING**

$= \frac{27\pi}{2}$ ← **½ PT FOR ANSWER**

Give the complete definition of the definite integral. **NO PARTIAL CREDIT.** **SCORE: ___ / 2 POINTS**

SEE DEFINITIONS AND THEOREMS HANDOUT FOR CORRECT ANSWER

A spherical tank of radius 10 feet is filled with water (density = 62.4 lb/ft^3). Write an integral for the work done in pumping half the water out of the top of the tank. **DO NOT EVALUATE THE INTEGRAL.** **SCORE: ___ / 4 POINTS**

THERE ARE MANY DIFFERENT WAYS TO WRITE THIS INTEGRAL, DEPENDING ON THE SCALE USED

FIND THE SCALE BELOW THAT MATCHES YOURS:
½ PT FOR CORRECT LOWER LIMIT OF INTEGRATION
½ PT FOR CORRECT UPPER LIMIT OF INTEGRATION
½ PT FOR 62.4
¼ PT FOR π
1 PT FOR CORRECT DISPLACEMENT FORMULA (THE FACTOR BELOW THAT COMES RIGHT AFTER 62.4π)
1 PT FOR CORRECT AREA FORMULA (THE FACTOR BELOW THAT COMES JUST BEFORE dx)
¼ PT FOR dx

If $x = 10$ is the top of the tank,
$x = 0$ is the middle,
$x = -10$ is the bottom,

then work = $\int_{0}^{10} 62.4\pi(10-x)(100-x^2)\,dx$

If $x = -10$ is the top of the tank,
$x = 0$ is the middle,
$x = 10$ is the bottom,

then work = $\int_{-10}^{0} 62.4\pi(10+x)(100-x^2)\,dx$

If $x = 0$ is the top of the tank,
$x = 10$ is the middle,
$x = 20$ is the bottom,

then work = $\int_{0}^{10} 62.4\pi(x)(100-(10-x)^2)\,dx$

or $\int_{0}^{10} 62.4\pi(x)(100-(x-10)^2)\,dx$

or $\int_{0}^{10} 62.4\pi(x)(20x-x^2)\,dx$

If $x = 20$ is the top of the tank,
$x = 10$ is the middle,
$x = 0$ is the bottom,

then work = $\int_{10}^{20} 62.4\pi(20-x)(100-(10-x)^2)\,dx$

or $\int_{10}^{20} 62.4\pi(20-x)(100-(x-10)^2)\,dx$

or $\int_{10}^{20} 62.4\pi(20-x)(20x-x^2)\,dx$

If $x = 0$ is the top of the tank,
$x = -10$ is the middle,
$x = -20$ is the bottom,

then work = $\int_{-10}^{0} 62.4\pi(-x)(100-(10+x)^2)\,dx$

or $\int_{-10}^{0} 62.4\pi(-x)(-20x-x^2)\,dx$

If $x = -20$ is the top of the tank,
$x = -10$ is the middle,
$x = 0$ is the bottom,

then work = $\int_{-20}^{-10} 62.4\pi(20+x)(100-(10+x)^2)\,dx$

or $\int_{-20}^{-10} 62.4\pi(20+x)(-20x-x^2)\,dx$