**Math 1B (7:30am – 8:20am)**
**Quiz 6**
**Thu Nov 12, 2009**

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __

**SCORE: ___ / 20 POINTS**

# NO CALCULATORS ALLOWED

$X$ is a continuous random variable with probability density function $p(x) = xe^x$ on $[0,1]$. **SCORE: ___ / 4 POINTS**
Find the mean value of $X$.

$$\int_0^1 x \cdot xe^x\,dx = \underbrace{\int_0^1 x^2 e^x\,dx}_{1} = \underbrace{x^2e^x}_{1} - \underbrace{2xe^x}_{1} + \underbrace{2e^x}_{\frac{1}{2}}\Big|_0^1$$

$$= (x^2-2x+2)e^x\Big|_0^1$$

$$= \underbrace{e-2}_{\frac{1}{2}}$$

| $u$ | | $dv$ |
|---|---|---|
| $x^2$ | + | $e^x$ |
| $2x$ | − | $e^x$ |
| $2$ | + | $e^x$ |
| $0$ | | $e^x$ |

Evaluate $\int x^3 (\ln x)^2\,dx$. **SCORE: ___ / 4 POINTS**

| $u$ | | $dv$ |
|---|---|---|
| $(\ln x)^2$ | + | $x^3$ |
| $2\ln x \cdot \frac{1}{x}$ | | $\frac{1}{4}x^4$ |
| $x$ — — — | | — $\frac{1}{x}$ |
| $2\ln x$ | − | $\frac{1}{4}x^3$ |
| $\frac{2}{x}$ | | $\frac{1}{16}x^4$ |
| $x$ — — — | | — $\frac{1}{x}$ |
| $2$ | + | $\frac{1}{16}x^3$ |
| $0$ | | $\frac{1}{64}x^4$ |

$$\underbrace{\frac{1}{4}x^4(\ln x)^2}_{1\frac{1}{2}} - \underbrace{\frac{1}{8}x^4\ln x}_{1\frac{1}{2}} + \underbrace{\frac{1}{32}x^4}_{\frac{3}{4}} + \underbrace{C}_{\frac{1}{4}}$$

Evaluate $\int \sin^{-1} x\,dx$. **SCORE: ___ / 4 POINTS**

| $u$ | | $dv$ |
|---|---|---|
| $\sin^{-1}x$ | + | $1$ |
| $\frac{1}{\sqrt{1-x^2}}$ | − | $x$ |

$$1\ \underbrace{x\sin^{-1}x}_{} - \underbrace{\int \frac{x}{\sqrt{1-x^2}}\,dx}_{\frac{1}{2}}$$

$u = 1-x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$$= x\sin^{-1}x + \underbrace{\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du}_{1}$$

$$= x\sin^{-1}x + \frac{1}{2}\,2\sqrt{u} + C$$

$$= \underbrace{x\sin^{-1}x}_{\frac{1}{2}} + \underbrace{\sqrt{1-x^2}}_{\frac{3}{4}} + \underbrace{C}_{\frac{1}{4}}$$

CONTINUED ON OTHER SIDE ➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔➔

Evaluate $\int e^{-4x}\sin 3x\,dx$.

SCORE: ___ / 4 POINTS

| $u$ | $dv$ |
| --- | --- |
| $\sin 3x$ | $+\ e^{-4x}$ |
| $3\cos 3x$ | $-\ -\frac{1}{4}e^{-4x}$ |
| $-9\sin 3x$ | $+\ \frac{1}{16}e^{-4x}$ |

$$= \underbrace{-\frac{1}{4}e^{-4x}\sin 3x}_{1} \underbrace{-\frac{3}{16}e^{-4x}\cos 3x}_{1} \underbrace{-\frac{9}{16}\int e^{-4x}\sin 3x\,dx}_{\frac{1}{2}}$$

SO,

$$\underbrace{\frac{25}{16}\int e^{-4x}\sin 3x\,dx = -\frac{1}{4}e^{-4x}\sin 3x - \frac{3}{16}e^{-4x}\cos 3x}_{\frac{3}{4}}$$

$$\int e^{-4x}\sin 3x\,dx = \underbrace{-\frac{4}{25}e^{-4x}\sin 3x - \frac{3}{25}e^{-4x}\cos 3x}_{\frac{1}{2}} \underbrace{+C}_{\frac{1}{4}}$$

Evaluate $\int \frac{3}{\sqrt{7-6x-x^2}}\,dx$.

SCORE: ___ / 4 POINTS

$$= \int \frac{3}{\sqrt{16-(x^2+6x+9)}}\,dx$$

$$= \int \frac{3}{\sqrt{\underbrace{16-(x+3)^2}_{1}}}\,dx$$

$$= \underbrace{\frac{3}{4}\int \frac{1}{\sqrt{1-\left(\frac{x+3}{4}\right)^2}}\,dx}_{1}$$

$u = \frac{x+3}{4}$

$du = \frac{1}{4}dx \Rightarrow dx = 4du$

$$= \underbrace{\frac{3}{4}\cdot 4\int \frac{1}{\sqrt{1-u^2}}\,du}_{1}$$

$$= 3\sin^{-1}u + C$$

$$= \underbrace{3\sin^{-1}\left(\frac{x+3}{4}\right)}_{\frac{3}{4}} \underbrace{+C}_{\frac{1}{4}}$$

**BONUS QUESTION** Evaluate $\int \frac{2x+1}{\sqrt{8-3x-2x^2}}\,dx$. <u>The answer will involve fractions and irrationals.</u>

SCORE: ___ / 4 POINTS

$$\int \frac{2x+1}{\sqrt{8-3x-2x^2}}\,dx = \int \frac{-\frac{1}{2}(-4x-3)-\frac{1}{2}}{\sqrt{8-3x-2x^2}}\,dx = \boxed{-\frac{1}{2}}\int \boxed{\frac{-4x-3}{\sqrt{8-3x-2x^2}}}\,dx - \frac{1}{2}\int \frac{1}{\sqrt{8-3x-2x^2}}\,dx$$

$$\boxed{\begin{aligned} u &= 8-3x-2x^2 \\ du &= (-4x-3)\,dx \end{aligned}}$$

$\frac{1}{2}$ POINT EACH

$$= -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du \boxed{-\frac{1}{2}}\int \boxed{\frac{1}{\sqrt{\frac{73}{8}-2\left(x+\frac{3}{4}\right)^2}}}\,dx$$

$$= -\frac{1}{\cancel{2}}\cancel{2}\sqrt{u} - \frac{1}{2}\int \boxed{\frac{1}{\sqrt{\frac{73}{8}}\sqrt{1-\frac{16}{73}\left(x+\frac{3}{4}\right)^2}}}\,dx$$

$$\boxed{\begin{aligned} u &= \frac{4}{\sqrt{73}}\left(x+\frac{3}{4}\right) \\ dx &= \frac{\sqrt{73}}{4}\,du \end{aligned}}$$

$$= \boxed{-\sqrt{8-3x-2x^2}} - \frac{1}{2}\int \boxed{\frac{\sqrt{73}}{4}}\,\boxed{\frac{\sqrt{8}}{\sqrt{73}}}\,\boxed{\frac{1}{\sqrt{1-u^2}}}\,du$$

$$= -\sqrt{8-3x-2x^2} - \frac{\sqrt{2}}{4}\sin^{-1}u + C$$

$$= -\sqrt{8-3x-2x^2} \boxed{-\frac{\sqrt{2}}{4}\sin^{-1}\frac{4}{\sqrt{73}}\left(x+\frac{3}{4}\right)} + C$$