Math 1B (7:30am – 8:20am)
Quiz 8
Fri Dec 4, 2009

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __

SCORE: ___ / 20 POINTS

NO CALCULATORS ALLOWED

Solve the following initial value problem. **SIMPLIFY YOUR ANSWER.** **SCORE: ___ / 4 POINTS**

$y' = \frac{y^2}{x}, \quad y(1) = 2$

$\frac{1}{y^2} dy = \frac{1}{x} dx$

$-\frac{1}{y} = \ln|x| + C$

$y = \frac{-1}{\ln|x| + C}$

$y(1) = \frac{-1}{\ln|1| + C} = \frac{-1}{C} = 2 \Rightarrow C = -\frac{1}{2}$

$y = \frac{-1}{\ln|x| - \frac{1}{2}}$

$y = \frac{2}{1 - 2\ln|x|}$

A function $f(x) \geq 0$ is a probability density function (pdf) on $[1, \infty)$ if $\int_1^\infty f(x)\,dx = 1$. **SCORE: ___ / 3 POINTS**

Find the value of k if $f(x) = \frac{k}{x^p}$ (where $p > 1$) is a pdf on $[1, \infty)$.

$\int_1^\infty \frac{k}{x^p} dx$

$= k \int_1^\infty x^{-p} dx$

$= k \lim_{N \to \infty} \frac{1}{-p+1} x^{-p+1} \Big|_1^N$

$= k \lim_{N \to \infty} \frac{1}{1-p} (N^{-p+1} - 1)$

$p > 1 \Rightarrow -p < -1 \Rightarrow -p + 1 < 0$

so $\lim_{N \to \infty} N^{-p+1} = 0$

$k \cdot \frac{1}{1-p} \cdot (-1) = 1$

$k = p - 1$

[MULTIPLE CHOICE] Consider the following statements: **SCORE: ___ / 2 POINTS**

#1: If $f(x)$ is continuous on $(0, 1]$ and $\lim_{x \to 0} f(x) = \infty$, then $\int_0^1 f(x)\,dx$ must diverge.

#2: If $f(x)$ is continuous on $[1, \infty)$ and $\lim_{x \to \infty} f(x) = 0$, then $\int_1^\infty f(x)\,dx$ must converge.

Which of the above statements is/are true ?

[A] Both #1 and #2 are true. **[B]** Only #1 is true. **[C]** Only #2 is true. **[D]** Both #1 and #2 are false.

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Evaluate the following integrals, if possible. **Use proper notation.** If an integral diverges, write DIVERGES. **SCORE: ___ / 7 POINTS**

[a] $\int_{-1}^{1} \frac{x}{x^2-4}\,dx$ CONTINUOUS ON $[-1,1]$

$u = x^2 - 4$ $\begin{cases} x = 1 \rightarrow u = -3 \\ x = -1 \rightarrow u = -3 \end{cases}$ $\int_{-3}^{-3} \frac{1}{2}\frac{1}{u}\,du = 0$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

[b] $\int_{0}^{\infty} \frac{1}{\sqrt{x}\,e^{\sqrt{x}}}\,dx = \int_0^1 \frac{1}{\sqrt{x}\,e^{\sqrt{x}}}\,dx + \int_1^\infty \frac{1}{\sqrt{x}\,e^{\sqrt{x}}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$\int \frac{1}{\sqrt{x}\,e^{\sqrt{x}}}\,dx$

$= 2\int \frac{1}{e^u}\,du$

$= -2e^{-u} = -2e^{-\sqrt{x}}$

$= \lim_{N\to0^+} -2e^{-\sqrt{x}}\Big|_N^1 + \lim_{M\to\infty} -2e^{-\sqrt{x}}\Big|_1^M$

$= \lim_{N\to0^+}(-2e^{-1} + 2e^{-\sqrt{N}}) + \lim_{M\to\infty}(-2e^{-\sqrt{M}} + 2e^{-1})$

$= -2e^{-1} + 2 + 0 + 2e^{-1}$

$= 2$

Determine if $\int_1^\infty \frac{1}{x^2+e^{-x}}\,dx$ converges or diverges. Show supporting work/logic. **SCORE: ___ / 4 POINTS**

$0 \le \frac{1}{x^2+e^{-x}} \le \frac{1}{x^2}$ ON $[1,\infty)$

$\int_1^\infty \frac{1}{x^2}\,dx$ CONVERGES SINCE $p = 2 \ge 1$

SO, $\int_1^\infty \frac{1}{x^2+e^{-x}}\,dx$ CONVERGES