Complex fractions are fractions in which the numerator or denominator, or both, contain fractions. They are critical to analytic trigonometry and identity proving, because each individual trigonometric function can be expressed as a fraction involving other trigonometric functions. **Whenever a complex fraction appears, you should simplify it immediately, before you use it in any other operation.** Otherwise, you risk making a straightforward problem exponentially more difficult.

There are 2 primary ways to simplify complex fractions.

METHOD 1: Write the numerator and denominator each as a single fraction, then perform the implied division.

$$\frac{1-\frac{1}{c}}{\frac{1}{c}+\frac{1}{s}}$$

$$=\frac{\frac{c}{c}-\frac{1}{c}}{\frac{s}{cs}+\frac{c}{cs}}$$ Find the LCD for the numerator (c), and find the LCD for the denominator (cs)

$$=\frac{\frac{c-1}{c}}{\frac{s+c}{cs}}$$ Perform the addition/subtraction in the numerator and denominator

$$=\frac{c-1}{c}\times\frac{cs}{s+c}$$ Multiply by the reciprocal of the denominator

$$=\frac{s(c-1)}{s+c}$$ Cancel and collect remaining factors

METHOD 2: Multiply the numerator and denominator by the LCD of all the component fractions.

$$\frac{1-\frac{1}{c}}{\frac{1}{c}+\frac{1}{s}}$$

$$=\frac{1-\frac{1}{c}}{\frac{1}{c}+\frac{1}{s}}\times\frac{cs}{cs}$$ Find the LCD of $\frac{1}{c}$ (from numerator), $\frac{1}{c}$ and $\frac{1}{s}$ (from denominator)

$$=\frac{cs-s}{s+c}$$ Multiply the LCD into the numerator and denominator, using distribution as needed

$$=\frac{s(c-1)}{s+c}$$ Factor the numerator and denominator, and cancel if needed

You should try **both**, and decide which is more natural for you.

Simplify the following.
NOTE: c and s represent variables. **Do NOT assume they mean** $\cos x$ **or** $\sin x$ **.**

[a]	$\dfrac{c}{1+\frac{1}{c}}$	[b]	$\dfrac{\frac{s}{c}+1}{1-\frac{1}{c}}$	[c]	$\dfrac{\frac{1}{s}+1}{\frac{1}{s}-1}$	[d]	$\dfrac{\frac{c}{s}-1}{1-\frac{s}{c}}$	[e]	$\dfrac{\frac{s}{c}-\frac{c}{s}}{\frac{1}{s}+\frac{1}{c}}$
[f]	$1-\dfrac{s}{\frac{1}{s}+1}$	[g]	$\dfrac{\frac{c}{s}-1}{1+\frac{1}{s}}-c$	[h]	$c+\dfrac{\frac{1}{c}-1}{\frac{1}{c}+1}$	[i]	$\dfrac{\frac{s}{c}+1}{1+\frac{c}{s}}-1$	[j]	$1+\dfrac{\frac{1}{c}-\frac{1}{s}}{\frac{c}{s}-\frac{s}{c}}$

Some possible answers to choose from:
NOTE: half of these answers are not correct of any of the problems above

[1]	$\dfrac{c-s}{s}$	[2]	$\dfrac{c-s+1}{c-s}$	[3]	$\dfrac{1+s-s^2}{s+1}$	[4]	$\dfrac{-s(1+c)}{s+1}$	[5]	$1+c$
[6]	$\dfrac{s+1}{s-1}$	[7]	$\dfrac{c}{s}$	[8]	$c+s$	[9]	$\dfrac{c^2}{c+1}$	[10]	$\dfrac{s+c}{1-c}$
[11]	$s-c$	[12]	$\dfrac{c}{c+1}$	[13]	$\dfrac{s+c}{c-1}$	[14]	$\dfrac{1+s}{1-s}$	[15]	$\dfrac{s}{c}$
[16]	$-c$	[17]	$\dfrac{c^2+1}{1+c}$	[18]	$\dfrac{s-c}{c}$	[19]	$\dfrac{c+s-1}{c+s}$	[20]	$\dfrac{1-s+s^2}{1+s}$