Math 51 (7:30am – 8:20am)
Midterm 3
Wed Jun 10, 2009

SCORE: ___ / 140 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
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[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED ON THIS SECTION

Fill in the blanks.

SCORE: ___ / 12 POINTS

[a] $\arccos(-1) =$ π

[b] $\sin^{-1}\frac{\sqrt{3}}{2} =$ $\frac{\pi}{3}$

[c] $\arctan(-\sqrt{3}) =$ $-\frac{\pi}{3}$

[d] $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ $\frac{3\pi}{4}$

Solve $7\pi - 6\arccos x = 2\pi$.

SCORE: ___ / 12 POINTS

$-6\arccos x = -5\pi$

$\arccos x = \frac{5\pi}{6}$

$x = \cos\frac{5\pi}{6}$

$x = -\frac{\sqrt{3}}{2}$

Find the exact value of $6\cos 75^\circ \sin 75^\circ$.

SCORE: ___ / 12 POINTS

$= 3(2\sin 75^\circ \cos 75^\circ)$

$= 3\sin 2(75^\circ)$

$= 3\sin 150^\circ$

$= 3\cdot\frac{1}{2} = \frac{3}{2}$

Solve $1 + 2\sin\frac{x}{2} = 0$.

SCORE: ___ / 12 POINTS

$\sin\frac{x}{2} = -\frac{1}{2}$

$\frac{x}{2} = \frac{7\pi}{6} + 2n\pi$ OR $\frac{11\pi}{6} + 2n\pi$

$x = \frac{7\pi}{3} + 4n\pi$ OR $\frac{11\pi}{3} + 4n\pi$

Prove the identity $\dfrac{1-\cos 2x}{1-\cos x} = 2+2\cos x$.

SCORE: ___ / 14 POINTS

$$\frac{1-\cos 2x}{1-\cos x}$$

$$= \frac{1-(2\cos^2 x - 1)}{1-\cos x}$$

$$= \frac{2-2\cos^2 x}{1-\cos x}$$

$$= \frac{2(1-\cos^2 x)}{1-\cos x}$$

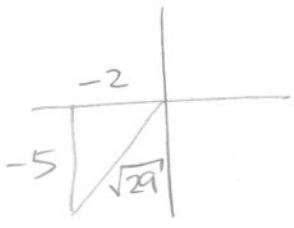
$$= \frac{2(1+\cos x)(1-\cos x)}{1-\cos x}$$

$$= 2(1+\cos x)$$

$$= 2+2\cos x \quad \text{QED}$$

If $\cos x = -\dfrac{2}{\sqrt{29}}$ and $\pi < x < \dfrac{3\pi}{2}$, find $\tan 2x$.

SCORE: ___ / 12 POINTS



$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$= \frac{2\left(\frac{5}{2}\right)}{1-\left(\frac{5}{2}\right)^2}$$

$$= \frac{5}{1-\frac{25}{4}}$$

$$= \frac{5}{-\frac{21}{4}} = 5\cdot\left(-\frac{4}{21}\right) = \frac{-20}{21}$$

Find the exact value of $\cos\dfrac{13\pi}{12}$.

SCORE: ___ / 12 POINTS

$$\cos\frac{13\pi}{12} = \cos\left(\frac{4\pi}{12}+\frac{9\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)$$

$$= \cos\frac{\pi}{3}\cos\frac{3\pi}{4} - \sin\frac{\pi}{3}\sin\frac{3\pi}{4}$$

$$= \frac{1}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{-\sqrt{2}-\sqrt{6}}{4}$$

Solve $\cos 2x + \cos x = -1$ in $[0, 2\pi)$.

SCORE: ___ / 12 POINTS

$2\cos^2 x - 1 + \cos x = -1$

$2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0$ OR $2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

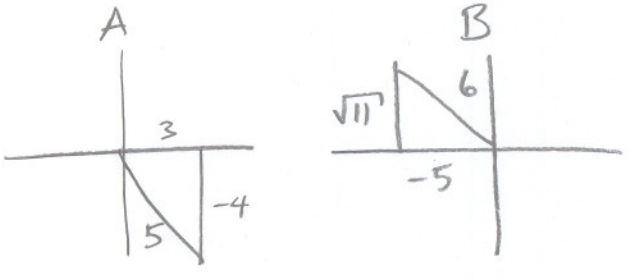
$x = \frac{\pi}{2}, \frac{3\pi}{2}$ OR $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad x = \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$

Find the exact value of $\sin\left(\tan^{-1}\left(-\frac{4}{3}\right) - \cos^{-1}\left(-\frac{5}{6}\right)\right)$.

SCORE: ___ / 16 POINTS

LET $A = \tan^{-1}\left(-\frac{4}{3}\right)$

AND $B = \cos^{-1}\left(-\frac{5}{6}\right)$



$\sin(A-B)$

$= \sin A\cos B - \cos A \sin B$

$= \frac{-4}{5}\cdot\frac{-5}{6} - \frac{3}{5}\cdot\frac{\sqrt{11}}{6}$

$= \frac{20 - 3\sqrt{11}}{30}$

Prove the identity $\frac{\cos\theta}{\sec\theta + \tan\theta} = 1 - \sin\theta$.

SCORE: ___ / 14 POINTS

$\frac{\cos\theta}{\sec\theta + \tan\theta}$

$= \frac{\cos\theta(\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$

$= \frac{\cos\theta\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)}{\sec^2\theta - \tan^2\theta}$

$= \frac{1 - \sin\theta}{1}$

$= 1 - \sin\theta$ QED

HAND IN THIS SECTION BEFORE YOU PICK UP YOUR CALCULATOR

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CALCULATORS ALLOWED ON THIS SECTION

Solve $1 - 2\cot x = 4$ in $[0, 2\pi)$.

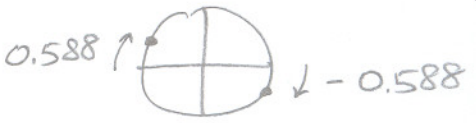
SCORE: ___ / 12 POINTS

$\cot x = -\frac{3}{2}$

$\tan x = -\frac{2}{3}$ $\qquad \tan^{-1}\left(-\frac{2}{3}\right) = -0.588$

$x = 2\pi + -0.588 = 5.695$

OR $\quad \pi - 0.588 = 2.554$



BONUS QUESTION

Find (and simplify) a formula for $\tan 3t$ in terms of $\tan t$.

SCORE: ___ / 14 POINTS