Math 51 (9:30am – 10:20am)
Midterm 3
Wed Jun 10, 2009

SCORE: ___ / 140 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
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NO CALCULATORS ALLOWED ON THIS SECTION

Fill in the blanks. **SCORE: ___ / 12 POINTS**

[a] $\cos^{-1} 0 =$ $\frac{\pi}{2}$

[b] $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) =$ $-\frac{\pi}{6}$

[c] $\arcsin\frac{1}{2} =$ $\frac{\pi}{6}$

[d] $\arccos\left(-\frac{1}{2}\right) =$ $\frac{2\pi}{3}$

Solve $5\pi - 3\arctan x = 3\pi$. **SCORE: ___ / 12 POINTS**

$-3\arctan x = -2\pi$

$\arctan x = \frac{2\pi}{3}$ NO SOLUTION $\frac{2\pi}{3} > \frac{\pi}{2}$

Find the exact value of $8\sin^2 75^\circ - 4$. **SCORE: ___ / 12 POINTS**

$= -4(1 - 2\sin^2 75^\circ)$

$= -4\cos 2(75^\circ)$

$= -4\cos 150^\circ$

$= -4\left(-\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

Solve $1 + 2\cos\frac{x}{2} = 0$. **SCORE: ___ / 12 POINTS**

$\cos\frac{x}{2} = -\frac{1}{2}$

$\frac{x}{2} = \frac{2\pi}{3} + 2n\pi$ OR $\frac{4\pi}{3} + 2n\pi$

$x = \frac{4\pi}{3} + 4n\pi$ OR $\frac{8\pi}{3} + 4n\pi$

Prove the identity $\dfrac{\cos(x-y)}{\cos x \sin y} = \tan x + \cot y$.

SCORE: ___ / 14 POINTS

$$\frac{\cos(x-y)}{\cos x \sin y} = \frac{\cos x \cos y + \sin x \sin y}{\cos x \sin y}$$

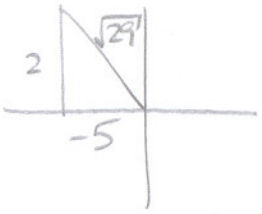
$$= \frac{\cos x \cos y}{\cos x \sin y} + \frac{\sin x \sin y}{\cos x \sin y}$$

$$= \cot y + \tan x$$

$$= \tan x + \cot y \quad \text{QED}$$

If $\sin x = \dfrac{2}{\sqrt{29}}$ and $\dfrac{\pi}{2} < x < \pi$, find $\tan 2x$.

SCORE: ___ / 12 POINTS



$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$= \frac{2\left(-\frac{2}{5}\right)}{1-\left(-\frac{2}{5}\right)^2}$$

$$= \frac{-\frac{4}{5}}{1-\frac{4}{25}}$$

$$= \frac{-\frac{4}{5}}{\frac{21}{25}}$$

$$= -\frac{4}{5} \cdot \frac{25}{21}$$

$$= \frac{-20}{21}$$

Find the exact value of $\cos\dfrac{17\pi}{12}$.

SCORE: ___ / 12 POINTS

$$\cos\frac{17\pi}{12} = \cos\left(\frac{8\pi}{12} + \frac{9\pi}{12}\right)$$

$$= \cos\left(\frac{2\pi}{3} + \frac{3\pi}{4}\right)$$

$$= \cos\frac{2\pi}{3}\cos\frac{3\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{3\pi}{4}$$

$$= -\frac{1}{2} \cdot -\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

Solve $\sin 2x = -\sin x$ in $[0, 2\pi)$.

SCORE: ___ / 12 POINTS

$2\sin x \cos x = -\sin x$

$2\sin x \cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$ OR $2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

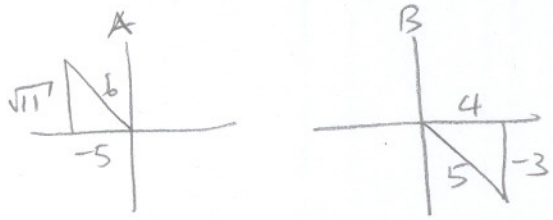
$x = 0, \pi$ OR $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\qquad x = 0, \frac{2\pi}{3}, \pi \frac{4\pi}{3}$

Find the exact value of $\sin\left(\cos^{-1}\left(-\frac{5}{6}\right) - \tan^{-1}\left(-\frac{3}{4}\right)\right)$.

SCORE: ___ / 16 POINTS

LET $A = \cos^{-1}\left(-\frac{5}{6}\right)$

AND $B = \tan^{-1}\left(-\frac{3}{4}\right)$



$\sin(A-B)$

$= \sin A \cos B - \cos A \sin B$

$= \frac{\sqrt{11}}{6} \cdot \frac{4}{5} - \left(-\frac{5}{6}\right)\left(\frac{-3}{5}\right)$

$= \frac{4\sqrt{11} - 15}{30}$

Prove the identity $\frac{\tan\theta}{\csc\theta - \cot\theta} = 1 + \sec\theta$.

SCORE: ___ / 14 POINTS

$\frac{\tan\theta}{\csc\theta - \cot\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} \cdot \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta}{(1-\cos\theta)\cos\theta}$

$= \frac{1-\cos^2\theta}{(1-\cos\theta)\cos\theta}$

$= \frac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$

$= \sec\theta + 1 = 1 + \sec\theta$ QED

HAND IN THIS SECTION BEFORE YOU PICK UP YOUR CALCULATOR

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CALCULATORS ALLOWED ON THIS SECTION

Solve $1 - 2\csc x = 4$ in $[0, 2\pi)$.

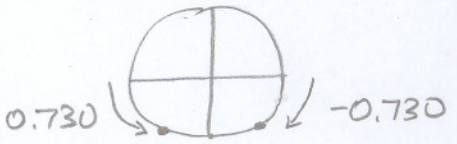
SCORE: ___ / 12 POINTS

$\csc x = -\frac{3}{2}$

$\sin x = -\frac{2}{3}$ $\qquad \sin^{-1}\left(-\frac{2}{3}\right) = -0.730$

$x = -0.730 + 2\pi = 5.553$

OR $\pi + 0.730 = 3.871$



BONUS QUESTION

Find (and simplify) a formula for $\tan 4t$ in terms of $\tan t$.

SCORE: ___ / 14 POINTS