Regenerating the sum and difference identities for sine, cosine and tangent from the double angle formulae

It is important to remember the sum, difference and double angle identities for the 3 primary trigonometric functions, because they may appear unexpectedly in their expanded forms, and if you are not familiar with the formulae, then you likely won't recognize how they can be simplified.

However, it can take quite a lot of practice or memorization to incorporate the 9 formulae.

In reality, you only need to memorize 3 – the double angle formulae (section 5.5) – as long as you understand how they come from the sum of angle formulae. That way, you can regenerate the sum identities, and sign changes then give the difference identities.

The 3 double angle formulae you need to memorize are

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

From the double angle sine formula, we can regenerate the sum of angles sine formula:

$$\sin 2x = 2\sin x \cos x$$

$$\sin(x+x) = \sin x \cos x + \cos x \sin x$$

Write $2x$ as $x + x$;
write 2 $\sin x \cos x$ as the sum of 2 products
NOTE: **Since the left side starts with** $\sin(x\cdots$**,**
make sure the right side starts with $\sin x\cdots$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

Change the 2nd x into y throughout
and you have the sum of angles sine identity
NOTE: **The trigonometric functions in the first product are the cofunctions of the corresponding trigonometric functions in the second product**

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

Change + into – throughout
and you have the difference of angles sine identity

From the double angle cosine formula, we can regenerate the sum of angles cosine formula:

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos(x + x) = \cos x \cos x - \sin x \sin x$$

Write 2x as $x + x$;
write each square as a product
NOTE: **Since the left side starts with** $\cos(x\cdots$**,**
make sure the right side starts with $\cos x\cdots$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

Change the 2nd x into y throughout
and you have the sum of angles cosine identity
NOTE: **The trigonometric functions in the first product are the cofunctions of the corresponding trigonometric functions in the second product**

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

Change + into –, and – into +
and you have the difference of angles cosine identity

From the double angle tangent formula, we can regenerate the sum of angles tangent formula:

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\tan(x + x) = \frac{\tan x + \tan x}{1 - \tan x \tan x}$$

Write 2x as $x + x$;

write 2 tan x as the sum of 2 tangents;
write the square as a product

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Change the 2nd x into y throughout

and you have the sum of angles tangent identity

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Change + into –, and – into + throughout

and you have the difference of angles tangent identity