3.6: To graph a function $f(x)$ from its formula:

[1] Find the domain and discontinuities of $f(x)$.

[2] Find the y-intercepts (set $x = 0$) and x-intercepts (solve $f(x) = 0$) (if possible)

[3] At each discontinuity $x = a$ of $f(x)$, find $\lim_{x \to a+} f(x)$ and $\lim_{x \to a-} f(x)$ to determine whether there is a removable discontinuity, a jump discontinuity, an asymptote, or none of the above, at a.

Removable discontinuity: $\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x)$ and both limits exist

Jump discontinuity: $\lim_{x \to a+} f(x) \neq \lim_{x \to a-} f(x)$ and both limits exist

Asymptote: $\lim_{x \to a+} f(x) = \pm\infty$ or $\lim_{x \to a+} f(x) = \pm\infty$

[4] Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ to determine if there are any horizontal asymptotes.

[5] Solve $f'(x) = 0$ or undefined, and $f''(x) = 0$ or undefined within the domain, and find the values of $f(x)$ at those points.

[6] For each point in [5] where $f'(x)$ is undefined, determine if $\lim_{x \to a+} f'(x) = \pm\infty$ or $\lim_{x \to a-} f'(x) = \pm\infty$ to find vertical tangent lines.

[7] Draw a number line subdivided into intervals by the points in [3] and [5], and mark all discontinuities in [3] by type, all horizontal tangent lines in [5] ($f'(x) = 0$) and all vertical tangent lines in [6].

[8] Find the signs of $f'(x)$ and $f''(x)$ within each interval.

[9] Analyze the sign changes of $f'(x)$ and $f''(x)$ between each pair of intervals, and mark all inflection points and all local extrema by type.

[10] Analyze the signs of $f'(x)$ and $f''(x)$ within each interval, and sketch a representative shape based on concavity and direction (increasing/decreasing).

[11] Plot the asymptotes in [3] and [4], the limit points in [3], the points in [2] and [5], and connect using the shapes in [10].

SUMMARY: find domain, discontinuities, intercepts, asymptotes
number line = discontinuities, $f'(x)$ *or* $f''(x) = 0$ or undefined
perform sign analysis on $f'(x)$ and $f''(x)$ and sketch segments
find horizontal/vertical tangent lines, local max/min, inflection points
connect

3.7 & 3.8: For related rates and optimization problems, you must distinguish between a quantity, an amount and a unit.

A quantity is a property of an object (typically) that can be measured and assigned a numerical value. A quantity should usually not include any numbers or units.

An amount is the number that represents a measurement of a quantity.

A unit is the scale given to an amount.

Examples

If a person is 6 feet tall
the quantity is the height of the person

the amount is 6	or the amount is 72	or the amount is 2
the unit is feet	and the unit is inches	and the unit is yards

If a ball is falling at 30 feet per second
the quantity is the speed of the ball

the amount is 30	or the amount is 20 5/11
the unit is feet per second	and the unit is miles per hour

Notice that the amount and unit in a single situation can have multiple values, but the quantity is unique.

What are the quantities, amounts and units in the following situations ?
Find another amount and unit pair that is equivalent to the ones mentioned.

A bag of rice is 50 pounds.

A haircut is $15.

A storage shed holds 400 cubic feet.

A ladder is 10 feet.

A diver descends 25 meters.

A piece of cloth is 2 square feet.

The movie lasted 2 hours.

The movie started at 7pm.

My aunt just turned 50.

The Tropic of Cancer is located at 23° north.

The ferris wheel goes around once every 3 minutes.

3 square feet of wrapping paper are needed to wrap a gift box.

The engine is turning at 3000 rpm.

The odometer reads 35000 kilometers.

LA is 347 miles from SF.

3.7: Optimization Problems

[1] Draw a diagram, if possible.

[2] Name the quantity you want the largest/maximum or smallest/minimum value of.

[3] Name the quantities you can change to try to get the maximum or minimum value of [2].

[4] Find the formula for the quantity in [2] in terms of the quantity in [3].

IF THE FORMULA IN [4] INVOLVES MORE THAN ONE VARIABLE FROM [3].

[4a] Find any restrictions/relationships between the quantities in [3].

[4b] Solve [4a] in terms of one quantity.

[4c] Rewrite [4] in terms of one quantity using [4b].

[5] Find the smallest and largest allowable values of the independent quantity in [4].

[6] Use calculus to find the global maximum/minimum of the dependent quantity in [4] over the domain in [5].

3.8: Related Rates Problems

[1] Draw a diagram, if possible. **DO NOT LABEL ANY NUMBERS ON THE DIAGRAM, UNLESS THE QUANTITIES THEY REPRESENT NEVER CHANGE.**

[2] Identify any quantities whose rate of change information is given or can be calculated directly by differentiation or other means.

[3] Identify the quantity whose rate of change you want, and under what circumstances.

[4] Find an equation connecting the quantities in [2] and [3].

[5] Differentiate [4] implicitly with respect to time.

[6] Substitute all known information (you may need to use [4]).

[7] Solve for the desired rate of change.