Math 1A (9:30am – 10:20am)
Group Quiz 2
Wed Jan 21, 2008

Group Members' Names: ______________________
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SCORE: ___ / 10 POINTS

NO CALCULATORS ALLOWED

Use the Squeeze Theorem to prove that $\lim_{x \to 0} x^2 \tan^{-1} \frac{1}{x} = 0$. **NOTE: This limit can be proven without the Squeeze Theorem, but you are required to use the Squeeze Theorem to get credit.**

SCORE: ___ / 3 POINTS

SINCE $\boxed{-\frac{\pi}{2} < \tan^{-1}\frac{1}{x} < \frac{\pi}{2}}$ $\frac{1}{2}$

THEREFORE $\boxed{-\frac{\pi}{2}x^2 < x^2\tan^{-1}\frac{1}{x} < \frac{\pi}{2}x^2}$ $\frac{1}{2}$ FOR $x \neq 0$

ALSO $\boxed{\lim_{x \to 0} -\frac{\pi}{2}x^2 = \lim_{x \to 0} \frac{\pi}{2}x^2 = 0}$ 1

SO $\boxed{\lim_{x \to 0} x^2\tan^{-1}\frac{1}{x} = 0}$ $\boxed{\text{BY THE SQUEEZE THEOREM}}$

$\frac{1}{2}$ $\frac{1}{2}$

Find the discontinuities of $f(x) = \begin{cases} 3 - x & \text{if } x < -3 \\ x + 9 & \text{if } -3 < x < 2 \\ 5x + 2 & \text{if } x \geq 2 \end{cases}$ and state whether each discontinuity is removable or non-removable. **Show supporting algebraic work.**

SCORE: ___ / 3 POINTS

$\boxed{x = -3 \text{ IS A DISCONTINUITY}}$ $\frac{1}{4}$

$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} (3 - x) = 6$ $\frac{1}{4}$

$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} (x + 9) = 6$ $\frac{1}{4}$

$\lim_{x \to -3} f(x) = 6$ (LIMIT EXISTS) $\frac{1}{4}$

$\boxed{x = -3 \text{ IS A REMOVABLE DISCONTINUITY}}$ $\frac{1}{2}$

$\frac{1}{4}$ $\boxed{x = 2 \text{ IS A DISCONTINUITY}}$

$\frac{1}{4}$ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x + 9) = 11$

$\frac{1}{4}$ $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (5x + 2) = 12$

$\frac{1}{4}$ $\lim_{x \to 2} f(x)$ DNE

$\frac{1}{2}$ $\boxed{x = 2 \text{ IS NOT A REMOVABLE DISCONTINUITY}}$

QUESTIONS ON OTHER SIDE

Find the values of a and b which make $f(x)=\begin{cases} x+a & \text{if } x\le 1 \\ b-x & \text{if } 1<x<3 \\ x^2 & \text{if } x\ge 3 \end{cases}$ continuous. If no such values exist, **SCORE: ___ / 2 POINTS**

write IMPOSSIBLE. **<u>Show supporting algebraic work.</u>**

$\lim\limits_{x\to 1^-} f(x) = 1+a$ $\qquad$ $\lim\limits_{x\to 3^-} f(x) = b-3$

$\lim\limits_{x\to 1^+} f(x) = b-1$ $\qquad$ $\lim\limits_{x\to 3^+} f(x) = 9$

NEED $\boxed{1+a=b-1}$ AND $\boxed{b-3=9}$

SO $\boxed{b=12}$ $\qquad$ $\frac{1}{2}$ EACH

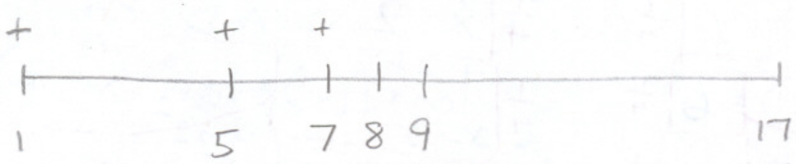
AND $1+a=11$

SO $\boxed{a=10}$

A table of values for a continuous function f are given below. **SCORE: ___ / 2 POINTS**

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$f(x)$	5	3	1	–1	–2	1	–2	3	–1	–3	–4	–1	2	3	1	2	–1	–2	1	4	–2

Use the method of bisections to find an interval of width 1 containing a zero of f starting with the interval $[1, 17]$.
Show the sequence of smaller intervals generated by the method.



$\boxed{[1,9]}$ $\frac{1}{2}$

$\boxed{[5,9]}$ $\frac{1}{2}$

$\boxed{[7,9]}$ $\frac{1}{2}$

$\boxed{[7,8]}$ $\frac{1}{2}$