Math 1A (9:30am – 10:20am)
Group Quiz 2
Wed Jan 21, 2008

Group Members' Names: ________________

SCORE: ___ / 10 POINTS

NO CALCULATORS ALLOWED

Use the Squeeze Theorem to prove that $\lim_{x\to 0} x^2 \tan^{-1}\frac{1}{x} = 0$. **NOTE: This limit can be proven without the Squeeze Theorem, but you are required to use the Squeeze Theorem to get credit.**

SCORE: ___ / 3 POINTS

SINCE $\boxed{-\frac{\pi}{2} < \tan^{-1}\frac{1}{x} < \frac{\pi}{2}}$ $\frac{1}{2}$

THEREFORE $\boxed{-\frac{\pi}{2}x^2 < x^2\tan^{-1}\frac{1}{x} < \frac{\pi}{2}x^2}$ $\frac{1}{2}$ FOR $x \neq 0$

ALSO $\boxed{\lim_{x\to 0} -\frac{\pi}{2}x^2 = \lim_{x\to 0}\frac{\pi}{2}x^2 = 0}$ 1

SO $\boxed{\lim_{x\to 0} x^2\tan^{-1}\frac{1}{x} = 0}$ $\boxed{\text{BY THE SQUEEZE TH'M}}$

$\frac{1}{2}$ $\frac{1}{2}$

Find the values of a and b which make $f(x) = \begin{cases} \cos x & \text{if } x < \pi \\ a + \sin x & \text{if } \pi \le x < 2\pi \\ b\tan x & \text{if } x \ge 2\pi \end{cases}$ continuous. If no such values exist, write IMPOSSIBLE. **Show supporting algebraic work.**

SCORE: ___ / 2 POINTS

$\lim_{x\to \pi^-} f(x) = -1$ $\qquad \lim_{x\to 2\pi^-} f(x) = a$

$\lim_{x\to \pi^+} f(x) = a$ $\qquad \lim_{x\to 2\pi^+} f(x) = 0$

NEED $\frac{1}{2}$ $\boxed{-1 = a}$ AND $\boxed{a = 0}$ $\frac{1}{2}$

$\boxed{\text{IMPOSSIBLE}}$

1

QUESTIONS ON OTHER SIDE

Find the discontinuities of $f(x)=\begin{cases} 3-x & \text{if } x<-3 \\ x+9 & \text{if } -3<x<2 \\ 5x+2 & \text{if } x\geq 2 \end{cases}$ and state whether each discontinuity is removable or non-removable. **Show supporting algebraic work.**

SCORE: ___ / 3 POINTS

$x=-3$ IS A DISCONTINUITY $\frac{1}{4}$

$\lim_{x\to -3^-} f(x) = 3--3 = 6$ $\frac{1}{4}$

$\lim_{x\to -3^+} f(x) = -3+9 = 6$ $\frac{1}{4}$

$\lim_{x\to -3} f(x)$ EXISTS $\frac{1}{4}$

SO, $x=-3$ IS A REMOVABLE DISCONTINUITY $\frac{1}{2}$

$\frac{1}{4}$ $x=2$ IS A DISCONTINUITY

$\frac{1}{4}$ $\lim_{x\to 2^-} f(x) = 2+9 = 11$

$\frac{1}{4}$ $\lim_{x\to 2^+} f(x) = 5(2)+2 = 12$

$\frac{1}{4}$ $\lim_{x\to 2} f(x)$ DNE

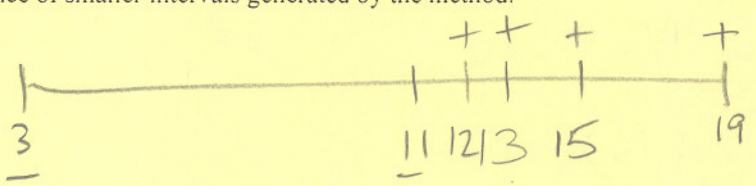
SO, $x=2$ IS NOT A $\frac{1}{2}$ REMOVABLE DISCONTINUITY

A table of values for a continuous function f are given below.

SCORE: ___ / 2 POINTS

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$f(x)$	5	3	1	–1	–2	1	–2	3	–1	–3	–4	–1	2	3	1	2	–1	–2	1	4	–2

Use the method of bisections to find an interval of width 1 containing a zero of f starting with the interval $[3, 19]$.
Show the sequence of smaller intervals generated by the method.



[11, 19] $\frac{1}{2}$

[11, 15] $\frac{1}{2}$

[11, 13] $\frac{1}{2}$

[11, 12] $\frac{1}{2}$