Math 1A (9:30am – 10:20am)
Group Quiz 8
Thu Mar 5, 2009

Group Members' Names: _____

SCORE: ___ / 10 POINTS

NO CALCULATORS ALLOWED

Let $f(x) = x^{\frac{4}{3}} - 2x^{\frac{2}{3}}$.

SCORE: ___ / 7 POINTS

[a] Find all critical numbers of $f(x)$.

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} - \frac{4}{3}x^{-\frac{1}{3}} = \frac{4}{3}x^{-\frac{1}{3}}(x^{\frac{2}{3}} - 1)$ IS UNDEFINED AT $x = 0$

$f'(x) = 0$ IF $x^{\frac{2}{3}} - 1 = 0$

$x^{\frac{2}{3}} = 1$

$x^2 = 1$

$x = \pm 1$

$\boxed{x = 0, \pm 1}$

[b] What does the SECOND DERIVATIVE TEST tell you about each critical number you found ?

$f''(x) = \frac{4}{9}x^{-\frac{2}{3}} + \frac{4}{9}x^{-\frac{4}{3}} = \frac{4}{9}x^{-\frac{4}{3}}(x^{\frac{2}{3}} + 1)$

SINCE $f'(0)$ IS UNDEFINED, THE 2ND DERIVATIVE TEST DOES NOT APPLY

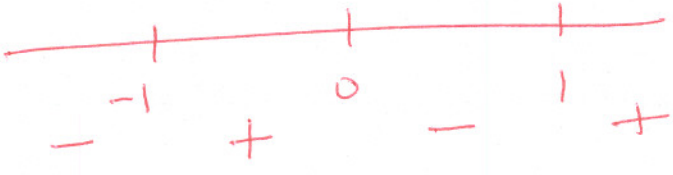
$f''(1) = \frac{4}{9}(1+1) > 0$
$f''(-1) = \frac{4}{9}(1+1) > 0$
} BOTH $x = \pm 1$ ARE LOCAL MIN'S.

[c] Find the intervals over which $f(x)$ is decreasing.

$\frac{4}{3}x^{-\frac{1}{3}} = \frac{4}{3\sqrt[3]{x}}$: $-$ | $-$ | $+$ | $+$

$x^{\frac{2}{3}} - 1$: $+$ | $-$ | $-$ | $+$

$(-\infty, -1] \cup [0, 1]$



f' : $-$ | $+$ | $-$ | $+$

[d] Find all inflection points of $f(x)$.

$f''(x) = \frac{4}{9}x^{-\frac{4}{3}}(x^{\frac{2}{3}} + 1)$ IS UNDEFINED AT $x = 0$ AND IS NEVER 0

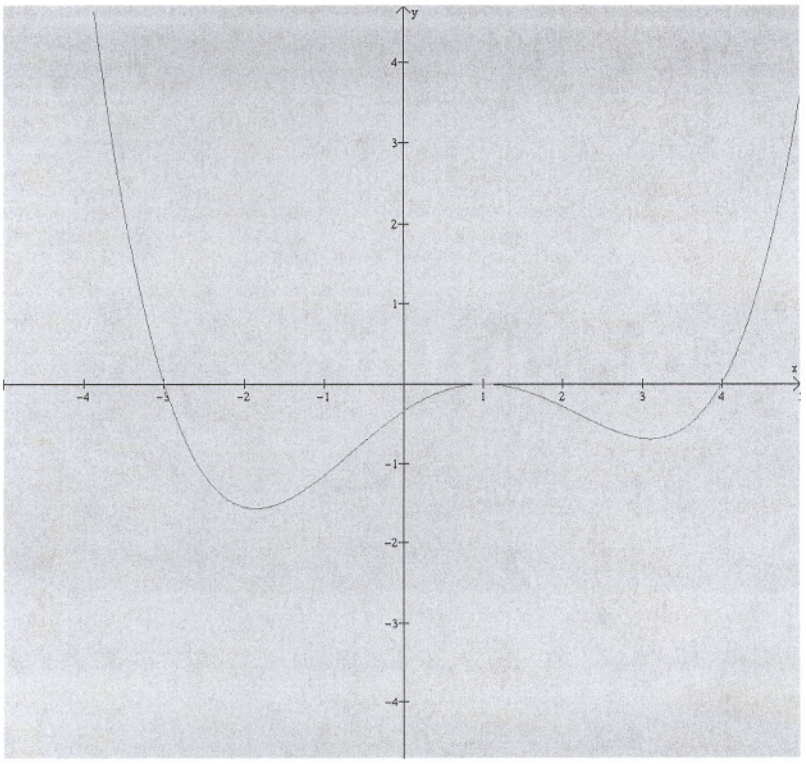
≥ 0 ≥ 1

$f''(x) > 0$ FOR ALL $x \neq 0$, SO NO I.P.

CONTINUED ON OTHER SIDE

The graph of $f'(x)$ is shown below. **NOTE: It is NOT the graph of $f(x)$.**

$f'(x)$



NOT $f(x)$

Answer the following questions about $f(x)$ without sketching $f(x)$. Explain **very briefly** why your answers are correct.

[a] Estimate the intervals over which $f(x)$ is decreasing.

$f'(x) \leq 0$ ON $[-3,4]$

[b] Estimate the x-coordinate(s) of all local maxima of $f(x)$.

$f'(x)$ CHANGES FROM >0 TO <0 AT $x=-3$

[c] Estimate the intervals over which $f(x)$ is concave up.

$f'(x)$ IS INCREASING ON $[-2,1]$ AND $[3,\infty)$