

SCORE: ____ / 100 POINTS

CALCULATOR NOT ALLOWED ON THIS SECTION

State the Squeeze Theorem.

SCORE: ____ / 5 POINTS

IF $f(x) \leq g(x) \leq h(x)$ ON (c, d) EXCEPT POSSIBLY
AT $x = a \in (c, d)$
AND $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$, THEN $\lim_{x \rightarrow a} g(x) = L$

State the precise definition of a limit.

SCORE: ____ / 5 POINTS

$\lim_{x \rightarrow a} f(x) = L$ IF FOR ALL $\epsilon > 0$, THERE IS A $\delta > 0$
SUCH THAT WHENEVER $0 < |x - a| < \delta$
WE HAVE $|f(x) - L| < \epsilon$

Determine the interval(s) on which $f(x) = x - \sqrt{9 - x^2}$ is continuous. Show all relevant work.

SCORE: ____ / 10 POINTS

$$\begin{aligned} \text{DOMAIN: } 9 - x^2 &\geq 0 \\ x^2 &\leq 9 \\ -3 &\leq x \leq 3 \\ [-3, 3] \end{aligned}$$

$$\begin{aligned} \text{SINCE } \lim_{x \rightarrow 3^-} x - \sqrt{9 - x^2} &= 3 - 0 = 3 \\ &= f(3) \\ \text{AND } \lim_{x \rightarrow -3^+} x - \sqrt{9 - x^2} &= -3 - 0 = -3 \\ &= f(-3) \\ f &\text{ IS CONTINUOUS ON } [-3, 3] \end{aligned}$$

Find all horizontal asymptotes of $f(x) = \frac{x}{\sqrt{x^2 - 9}}$. Show all relevant work.

SCORE: ____ / 12 POINTS

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 - 9}} &= \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 - 9}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x^2 - 9} \cdot \sqrt{\frac{1}{x^2}}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 - \frac{9}{x^2}}} = \frac{1}{\sqrt{1 - 0}} = 1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 - 9}} &= \lim_{x \rightarrow -\infty} \frac{1}{\sqrt{x^2 - 9} \cdot -\sqrt{\frac{1}{x^2}}} = \lim_{x \rightarrow -\infty} -\frac{1}{\sqrt{1 - \frac{9}{x^2}}} = \frac{-1}{\sqrt{1 - 0}} = -1 \end{aligned}$$

H.A. $y = \pm 1$

MULTIPLE CHOICE: $\lim_{x \rightarrow 0^-} e^{-\frac{1}{x}} =$

SCORE: ___ / 5 POINTS

[a] $-\infty$

[b] 0

[c] 1

[d] ∞

LETTER OF CORRECT ANSWER: [D]

Evaluate the following limits. Your answer should be a number, ∞ , $-\infty$ or "DNE" (if no other answer applies). SCORE: ___ / 25 POINTS
Show all relevant work as demonstrated in class. If an answer is "DNE", explain briefly.

[i] $\lim_{x \rightarrow -\infty} \sin(\tan^{-1} x) = -1$

$$\lim_{x \rightarrow -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

$$\lim_{x \rightarrow -\frac{\pi}{2}} \sin x = \sin(-\frac{\pi}{2}) = -1$$

[ii] $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 + 4x - 12} = \frac{0}{0}$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+6)}$$

$$= \lim_{x \rightarrow 2} \frac{x^2+2x+4}{x+6}$$

$$= \frac{2^2+2(2)+4}{2+6} = \frac{12}{8} = \frac{3}{2}$$

[iii] $\lim_{x \rightarrow 2} \frac{x+5}{\frac{6}{x+1}-3} = -7$

$$= \frac{2+5}{\frac{6}{2+1}-3}$$

$$= \frac{7}{2-3}$$

$$= -7$$

[iv] $\lim_{x \rightarrow 2} \frac{4-x^2}{x-\sqrt{x+2}} = \frac{0}{0}$

$$= \lim_{x \rightarrow 2} \frac{(4-x^2)(x+\sqrt{x+2})}{x^2-(x+2)}$$

$$= \lim_{x \rightarrow 2} \frac{(2-x)(2+x)(x+\sqrt{x+2})}{(x-2)(x+1)}$$

$$= \lim_{x \rightarrow 2} - \frac{(2+x)(x+\sqrt{x+2})}{x+1}$$

$$= - \frac{(2+2)(2+\sqrt{2+2})}{2+1} = -\frac{16}{3}$$

[v] $\lim_{x \rightarrow -1} f(x)$ where $f(x) = \begin{cases} 7-x^2 & \text{if } x < -1 \\ 3-4x & \text{if } -1 < x < 4 \\ x+2 & \text{if } x > 4 \end{cases}$ DNE

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (3-4x) = 7$$

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (7-x^2) = 6 \quad \neq$$