Math 1A (9:30am – 10:20am)
Midterm 1
Tue Jan 27, 2008

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your social security number ? __ __
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

CALCULATOR ALLOWED ON THIS SECTION

Sketch a graph of a function f with **all** the following properties. **SCORE: ___ / 10 POINTS**

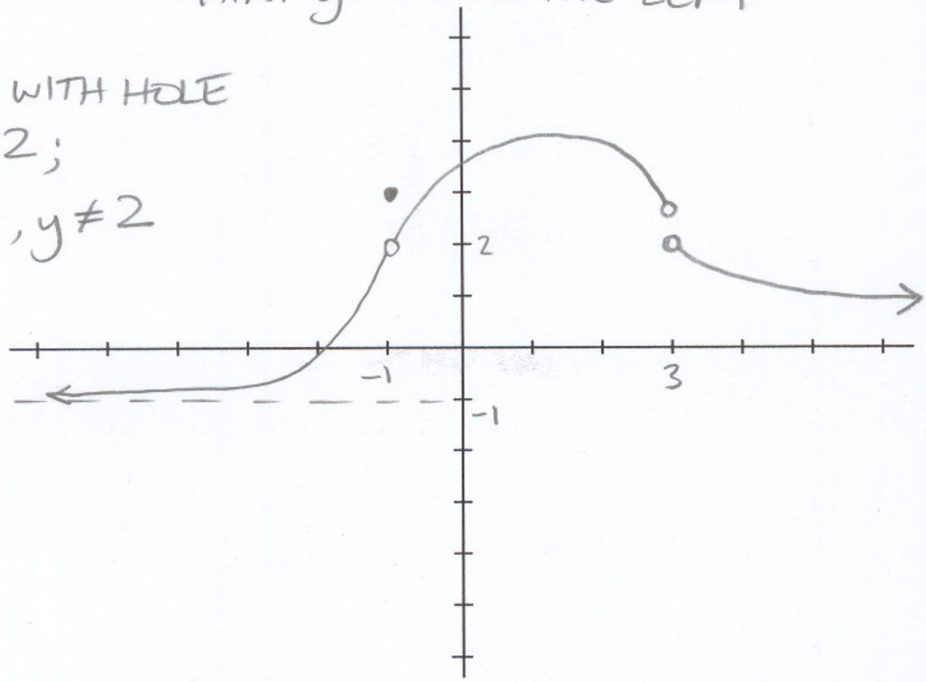
The domain of f is all real numbers except $x = 3$, ← NO VALUE AT x=3

f has a removable discontinuity at $x = -1$ and a non-removable discontinuity at $x = 3$, ← JUMP OR V.A. AT x=3

$\lim\limits_{x \to -1^-} f(x) = 2$, and $\lim\limits_{x \to -\infty} f(x) = -1$.

↑ H.A. y=−1 TO THE LEFT

2 SIDED LIMIT WITH HOLE AT x=−1, y=2;
DOT AT x=−1, y≠2



A function f is continuous from the right at $x = a$ if $\lim\limits_{x \to a^+} f(x) = f(a)$. **SCORE: ___ / 8 POINTS**

If $f(x) = \begin{cases} cx+3 & \text{if } x \le 2 \\ 5-x & \text{if } 2 < x < 3 \\ cx^2-4 & \text{if } x \ge 3 \end{cases}$, find all values of c so that f is continuous from the right at $x = 3$ (if possible).

<u>Show all relevant work.</u>

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (cx^2 - 4) = 9c - 4$$
$$f(3) = 9c - 4$$

EQUAL FOR ALL C

SO f IS CONT. FROM THE RIGHT AT x=3 FOR ALL C

Let $f(x) = 1 + x\cos 3x$. **SCORE: ___ / 12 POINTS**

[i] Prove that $f(x)$ has a zero in the interval $[0,16]$. **You must justify your argument properly as shown in class.**

f IS CONTINUOUS ON $[0,16]$ SINCE IT IS THE SUM, PRODUCT AND COMPOSITION OF FUNCTIONS THAT ARE CONTINUOUS EVERYWHERE

$f(0) > 0$

$f(16) < 0$

0 IS BETWEEN $f(0)$ AND $f(16)$

BY IVT, THERE IS A $c \in (0,16)$ SUCH THAT $f(c) = 0$

IE. c IS A ZERO OF f

[ii] **MULTIPLE CHOICE:** Use the method of bisections on the interval $[0,16]$ to find an interval of width 1 that contains a zero.

[a] $[7,8]$ [b] $[9,10]$ [c] $[11,12]$ [d] $[13,14]$

Is the statement below true or false ? If it is true, give a brief explanation why it is true. **SCORE: ___ / 8 POINTS**
If it is false, give a counterexample showing why it is false.

Statement: If $\lim_{x \to a} f(x)$ does not exist, then $\lim_{x \to a} \frac{1}{f(x)}$ does not exist

THE STATEMENT IS FALSE

LET $f(x) = \frac{1}{x^2}$ AND $a = 0$, SO $\frac{1}{f(x)} = x^2$

$\lim_{x \to 0} \frac{1}{x^2} = \infty$ IE. DNE

$\lim_{x \to 0} x^2 = 0$ DOES EXIST