Math 1A (9:30am – 10:20am)
Midterm 2
Mon Feb 23, 2008

SCORE: ___ / 100 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

CALCULATORS NOT ALLOWED

MULTIPLE CHOICE: The position of an object, in feet, at time t minutes is given by $s(t) = t^3 + 2t + 5$. Find the average velocity of the object from time $t = 1$ to $t = 3$. **SCORE: ___ / 5 POINTS**

[a] 10 ft/min [b] 11 ft/min [c] 15 ft/min [d] 16 ft/min

LETTER OF CORRECT ANSWER: [C] $\frac{s(3)-s(1)}{3-1}$

MULTIPLE CHOICE: The table below lists various values of $p(x)$ and $p'(x)$. **SCORE: ___ / 5 POINTS**
If $s(x) = \tan^{-1}(x - p(x))$, find the slope of the tangent line to $y = s(x)$ at $x = -2$.

x	–2	–1	0	1	2
$p(x)$	–1	2	–1	–3	1
$p'(x)$	3	–4	–2	3	5

[a] –1 [b] 0.5 [c] 0.6 [d] 0.8

LETTER OF CORRECT ANSWER: [A] $\frac{1}{1+(x-p(x))^2}(1-p'(x))$

State Rolle's Theorem. **SCORE: ___ / 5 POINTS**

IF f IS CONT. ON $[a,b]$ AND DIFF. ON (a,b) AND $f(a)=f(b)$
THEN $f'(c)=0$ FOR SOME $c \in (a,b)$

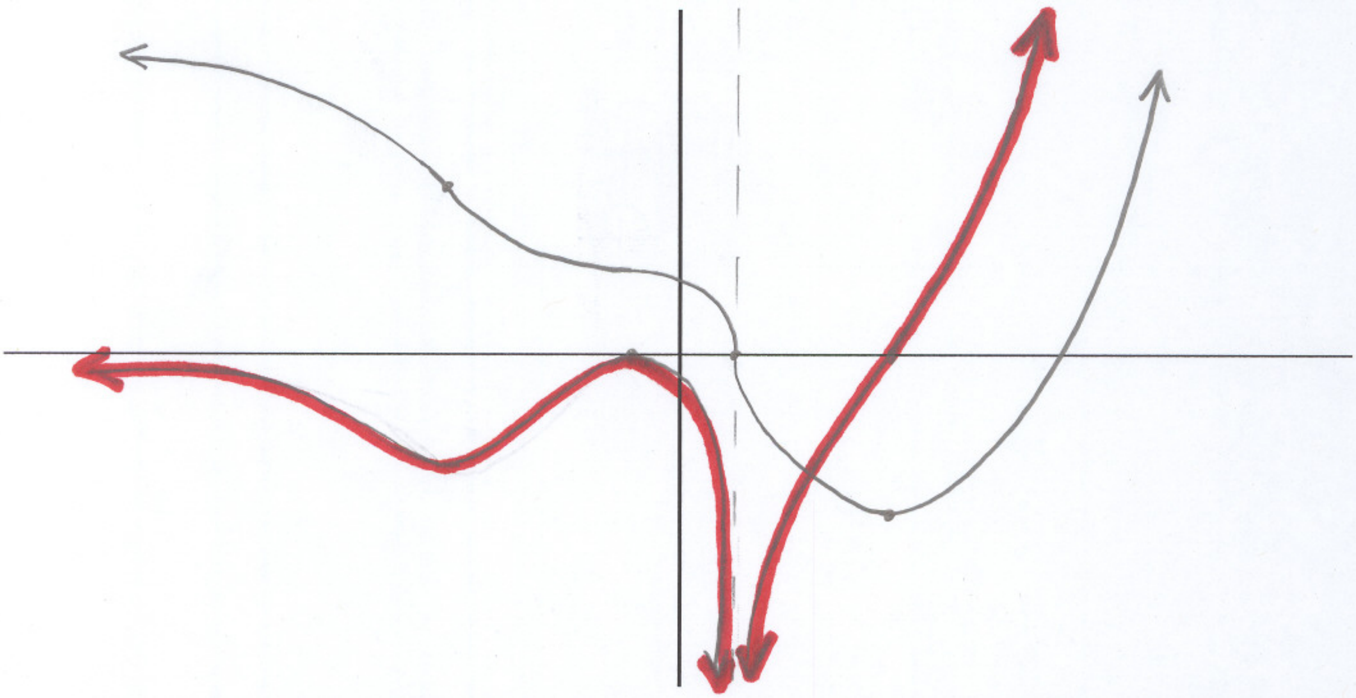
State the definition of instantaneous velocity. **SCORE: ___ / 5 POINTS**

IF $s(t)$ IS THE POSITION OF AN OBJECT (MOVING IN A STRAIGHT LINE)
AT TIME t,
THEN THE INSTANTANEOUS VELOCITY AT TIME $t=a$ IS $\lim_{h\to 0}\frac{s(a+h)-s(a)}{h}$

Temperature tends to be lower at higher elevations (heights) than at lower elevations. If $T(x)$ is the temperature (in degrees Celsius) at an elevation of x thousand feet, interpret the statement $T'(4) = -3$. **Specify the units of all relevant numbers.** **Do NOT use the words *instantaneous*, *slope* or *derivative*, nor the phrase *rate of change*. Do NOT use negative numbers in your answer.** **SCORE: ___ / 6 POINTS**

AT AN ELEVATION OF 4000 FEET,
THE TEMPERATURE DROPS 3°C PER 1000 FEET INCREASE IN
ELEVATION

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ on the same axes. **SCORE: ___ / 10 POINTS**



Consider the following 3 statements: **SCORE: ___ / 14 POINTS**

Statement 1: Rolle's Theorem applies to $f(x)=x^2+4x-3$ on the interval $[0, 4]$

Statement 2: The Mean Value Theorem applies to $f(x)=|x|$ on the interval $[-4, 4]$

Statement 3: The Mean Value Theorem applies to $f(x)=\frac{1}{x}$ on the interval $[2, 5]$

[i] Determine which of the three statements above is true, and give a brief explanation why it is true.
NOTE: Only one statement is true.

#3 : f IS CONT. ON [2,5] AND DIFF. ON (2,5)

[ii] Find the value of c guaranteed in the true statement in part [i].

$$f'(c)=\frac{f(5)-f(2)}{5-2}$$

$$-\frac{1}{c^2}=\frac{\frac{1}{5}-\frac{1}{2}}{5-2}=\frac{\frac{-3}{10}}{3}=-\frac{1}{10}$$

$$c^2=10$$

$$c=\pm\sqrt{10} \qquad c=\sqrt{10}\in(2,5)$$

If $f(x) = \dfrac{1}{x^2 - 1}$, find $f'(2)$ **using the definition of the derivative at a point**.

SCORE: ___ / 10 POINTS

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(2+h)^2 - 1} - \frac{1}{3}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{h^2+4h+3} - \frac{1}{3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3} - (h^2+4h+\cancel{3})}{3h(h^2+4h+3)}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}(h+4)}{3\cancel{h}(h^2+4h+3)} = \frac{-4}{3(3)} = \frac{-4}{9}$$

Determine which of the following 3 statements is true. Show all relevant work.
NOTE: Only one statement is true.

SCORE: ___ / 8 POINTS

Statement 1: $y = \ln(1 - x^4)$ is orthogonal to $y = -\dfrac{1}{8x^2} - \dfrac{x^2}{8}$

Statement 2: $y = \ln(1 - x^4)$ is orthogonal to $y = -\dfrac{8}{x^2} + \dfrac{x^2}{8}$

Statement 3: $y = \ln(1 - x^4)$ is orthogonal to $y = -\dfrac{8}{x^3} - \dfrac{x^2}{8}$

#1: $y = \ln(1 - x^4)$

$$\frac{dy}{dx} = \frac{1}{1-x^4} \cdot -4x^3$$

$$= \frac{4x^3}{x^4 - 1}$$

$y = -\dfrac{1}{8x^2} - \dfrac{x^2}{8}$

$$\frac{dy}{dx} = \frac{2}{8x^3} - \frac{2x}{8}$$

$$= \frac{1}{4x^3} - \frac{x}{4}$$

$$= \frac{1 - x^4}{4x^3}$$

$$\frac{4x^3}{x^4-1} \cdot \frac{1-x^4}{4x^3} = -1$$

Evaluate the following derivatives. **Simplify your anwers (unless otherwise stated).** **SCORE: ___ / 32 POINTS**

[a] Find $\dfrac{d^2y}{dt^2}$ if $y = \dfrac{4-5t-6t^3}{\sqrt[3]{t}}$.

$$y = 4t^{-\frac{1}{3}} - 5t^{\frac{2}{3}} - 6t^{\frac{8}{3}}$$

$$\frac{dy}{dt} = -\frac{4}{3}t^{-\frac{4}{3}} - \frac{10}{3}t^{-\frac{1}{3}} - 16t^{\frac{5}{3}}$$

$$\frac{d^2y}{dt^2} = \frac{16}{9}t^{-\frac{7}{3}} + \frac{10}{9}t^{-\frac{4}{3}} - \frac{80}{3}t^{\frac{2}{3}}$$

[b] Find $f'(x)$ if $f(x) = \dfrac{\cos x^3}{\sin^2 x}$.

$$f'(x) = \frac{(-\sin x^3)(3x^2)\sin^2 x - (\cos x^3)(2\sin x)\cos x}{(\sin^2 x)^2}$$

$$= \frac{-3x^2\sin^2 x \sin x^3 - 2\sin x \cos x \cos x^3}{\sin^4 x}$$

[c] Find $f'(x)$ if $f(x) = (\csc x)^{\cot x}$.

$$\ln f(x) = \cot x \ln \csc x$$

$$\frac{1}{f(x)}f'(x) = -\csc^2 x \ln \csc x + \cot x \frac{1}{\csc x}(-\csc x \cot x)$$

$$= -\csc^2 x \ln \csc x - \cot^2 x$$

$$f'(x) = f(x)(-\csc^2 x \ln \csc x - \cot^2 x)$$

$$= -(\csc x)^{\cot x}(\csc^2 x \ln \csc x + \cot^2 x)$$

[d] Find $g'(x)$ if $g(x) = \sin^{-1}\left(x^2 - [f(x)]^3\right)$ where $f(x)$ is an unspecified function. **DO NOT SIMPLIFY YOUR ANSWER.**

$$g'(x) = \frac{1}{\sqrt{1-(x^2-[f(x)]^3)^2}} \cdot (2x - 3[f(x)]^2 f'(x))$$

[e] Find $\dfrac{dy}{dx}$ if $\tan(2x-3y) - e^{-x^2} = xy^3$.

$$\sec^2(2x-3y)\cdot\left(2-3\frac{dy}{dx}\right) - e^{-x^2}(-2x) = y^3 + 3xy^2\frac{dy}{dx}$$

$$2\sec^2(2x-3y) - 3\sec^2(2x-3y)\frac{dy}{dx} + 2xe^{-x^2} = y^3 + 3xy^2\frac{dy}{dx}$$

$$2\sec^2(2x-3y) + 2xe^{-x^2} - y^3 = (3\sec^2(2x-3y) + 3xy^2)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2\sec^2(2x-3y) + 2xe^{-x^2} - y^3}{3\sec^2(2x-3y) + 3xy^2}$$