

SCORE: ____ / 100 POINTS

What month is your birthday ?

What are the first 2 digits of your address ?

What are the last 2 digits of your zip code ?

What are the last 2 digits of your social security number ?

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]**CALCULATORS NOT ALLOWED****MULTIPLE CHOICE:** The position of an object, in feet, at time t minutes is given by $s(t) = t^3 + 3t + 5$. SCORE: ____ / 5 POINTSFind the average velocity of the object from time $t = 1$ to $t = 3$.

[a] 10 ft/min

[b] 11 ft/min

[c] 15 ft/min

[d] 16 ft/min

LETTER OF CORRECT ANSWER: [D] $\frac{s(3)-s(1)}{3-1}$ **MULTIPLE CHOICE:** The table below lists various values of $p(x)$ and $p'(x)$. SCORE: ____ / 5 POINTSIf $s(x) = \tan^{-1}(x - p(x))$, find the slope of the tangent line to $y = s(x)$ at $x = -1$.

x	-2	-1	0	1	2
$p(x)$	-1	2	-1	-3	1
$p'(x)$	3	-4	-2	3	5

[a] -1

[b] 0.5

[c] 0.6

[d] 0.8

LETTER OF CORRECT ANSWER: [B] $\frac{1}{1+(x-p(x))^2(1-p'(x))}$

State Rolle's Theorem.

SCORE: ____ / 5 POINTS

SEE KEY +2t

State the definition of instantaneous velocity.

SCORE: ____ / 5 POINTS

SEE KEY +2t

Temperature tends to be lower at higher elevations (heights) than at lower elevations. If $T(x)$ is the temperature (in degrees Celsius) at an elevation of x thousand feet, interpret the statement $T'(4) = -3$. Specify the units of all relevant numbers.

SCORE: ____ / 6 POINTS

Do NOT use the words *instantaneous*, *slope* or *derivative*, nor the phrase *rate of change*. Do NOT use negative numbers in your answer.

SEE KEY +2t

Evaluate the following derivatives. Simplify your answers (unless otherwise stated).

SCORE: ____ / 32 POINTS

[a] Find $\frac{d^2 y}{dt^2}$ if $y = \frac{4 - 5t - 6t^3}{\sqrt[3]{t}}$.

[b] Find $f'(x)$ if $f(x) = \frac{\cos x^3}{\sin^2 x}$.

[c] Find $f'(x)$ if $f(x) = (\csc x)^{\cot x}$.

SEE
KEY
+2t

[d] Find $g'(x)$ if $g(x) = \sin^{-1}(x^2 - [f(x)]^3)$ where $f(x)$ is an unspecified function. DO NOT SIMPLIFY YOUR ANSWER.

[e] Find $\frac{dy}{dx}$ if $\tan(2x - 3y) - e^{-x^2} = xy^3$.

If $f(x) = \frac{1}{x^2 - 1}$, find $f'(2)$ using the definition of the derivative at a point.

SCORE: ____ / 10 POINTS

SEE KEY +2t

Determine which of the following 3 statements is true. Show all relevant work.

SCORE: ____ / 8 POINTS

NOTE: Only one statement is true.

Statement 1: $y = \ln(1 - x^4)$ is orthogonal to $y = -\frac{1}{8x^2} - \frac{x^2}{8}$

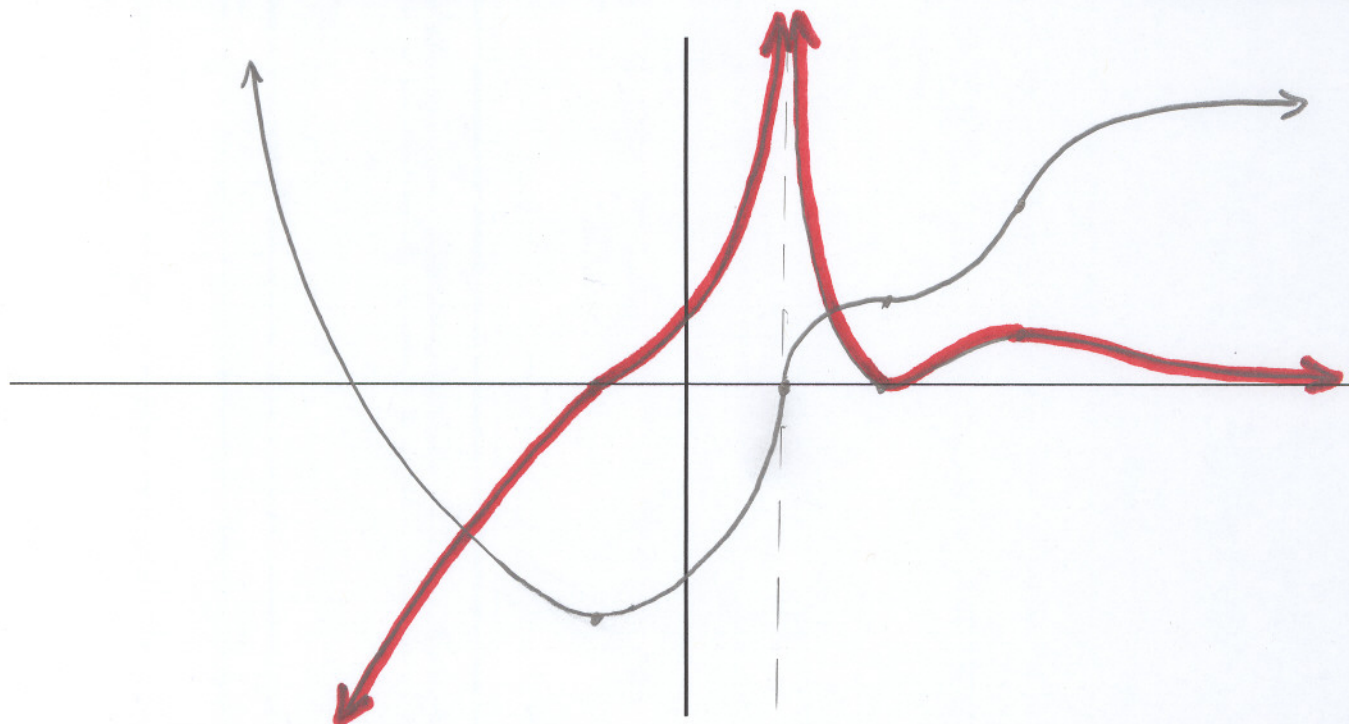
Statement 2: $y = \ln(1 - x^4)$ is orthogonal to $y = -\frac{8}{x^2} + \frac{x^2}{8}$

Statement 3: $y = \ln(1 - x^4)$ is orthogonal to $y = -\frac{8}{x^3} - \frac{x^2}{8}$

SEE KEY +2t

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ on the same axes.

SCORE: ___ / 10 POINTS



Consider the following 3 statements:

SCORE: ___ / 14 POINTS

Statement 1: Rolle's Theorem applies to $f(x) = x^2 + 4x - 3$ on the interval $[0, 3]$

Statement 2: The Mean Value Theorem applies to $f(x) = |x|$ on the interval $[-2, 2]$

Statement 3: The Mean Value Theorem applies to $f(x) = \frac{1}{x}$ on the interval $[2, 6]$

[i] Determine which of the three statements above is true, and give a brief explanation why it is true.

NOTE: Only one statement is true.

#3: f is CONT. ON $[2, 6]$ AND DIFF. ON $(2, 6)$

[ii] Find the value of c guaranteed in the true statement in part [i].

$$f'(c) = \frac{f(6) - f(2)}{6 - 2}$$

$$-\frac{1}{c^2} = \frac{\frac{1}{6} - \frac{1}{2}}{6 - 2} = \frac{-\frac{4}{12}}{4} = -\frac{1}{12}$$

$$c^2 = 12$$

$$c = \pm 2\sqrt{3}$$

$$c = 2\sqrt{3} \in (2, 6)$$