Math 1A (9:30am – 10:20am)
Midterm 2
Mon Feb 23, 2008

SCORE: ___ / 100 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

CALCULATORS NOT ALLOWED

MULTIPLE CHOICE: The position of an object, in feet, at time t minutes is given by $s(t) = t^3 - 2t + 5$. **SCORE: ___ / 5 POINTS**
Find the average velocity of the object from time $t = 1$ to $t = 3$.

[a] 10 ft/min [b] 11 ft/min [c] 15 ft/min [d] 16 ft/min

LETTER OF CORRECT ANSWER: [B] $\frac{s(3) - s(1)}{3 - 1}$

MULTIPLE CHOICE: The table below lists various values of $p(x)$ and $p'(x)$. **SCORE: ___ / 5 POINTS**
If $s(x) = \tan^{-1}(x + p(x))$, find the slope of the tangent line to $y = s(x)$ at $x = 2$.

x	–2	–1	0	1	2
$p(x)$	–1	2	–1	–3	1
$p'(x)$	3	–4	–2	3	5

[a] –1 [b] 0.5 [c] 0.6 [d] 0.8

LETTER OF CORRECT ANSWER: [C] $\frac{1}{1 + (x + p(x))^2}(1 + p'(x))$

State the definition of instantaneous velocity. **SCORE: ___ / 5 POINTS**

SEE KEY +2t

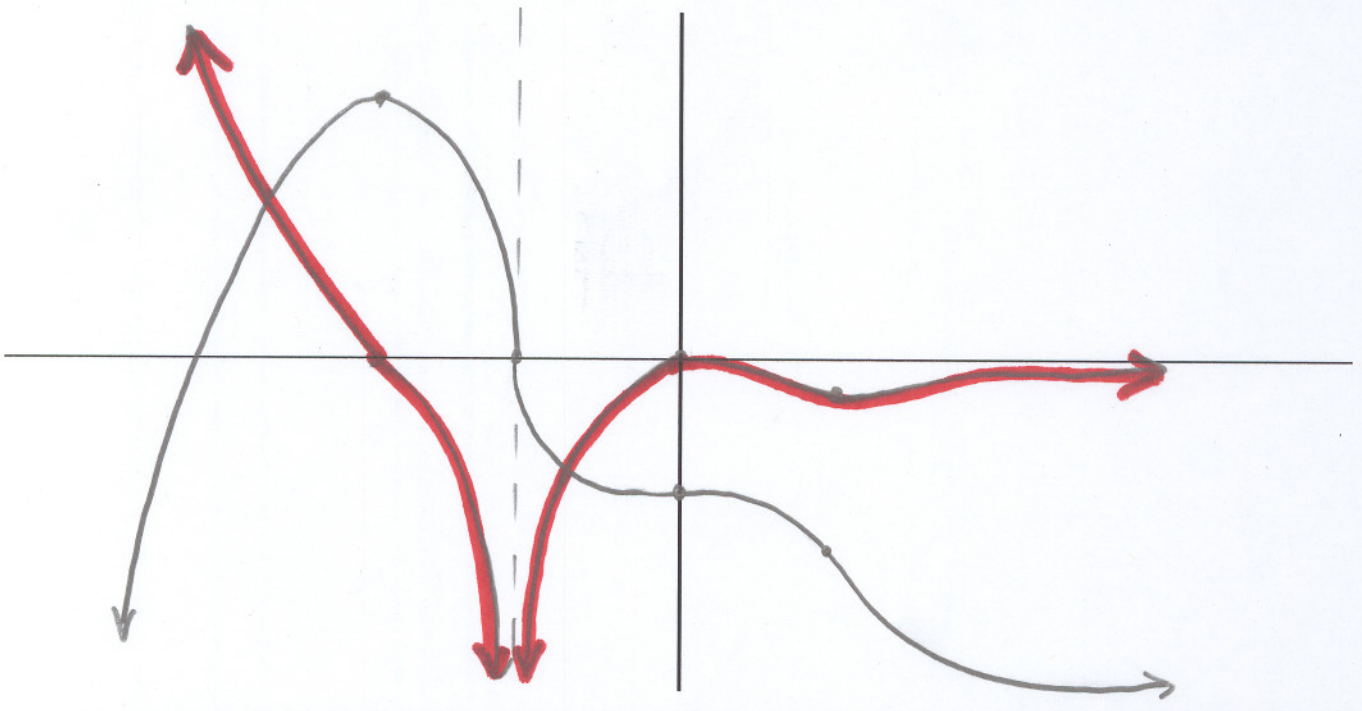
State Rolle's Theorem. **SCORE: ___ / 5 POINTS**

SEE KEY +2t

Temperature tends to be lower at higher elevations (heights) than at lower elevations. If $T(x)$ is the temperature (in degrees Celsius) at an elevation of x thousand feet, interpret the statement $T'(3) = -4$. **SCORE: ___ / 6 POINTS**
Specify the units of all relevant numbers.
Do NOT use the words *instantaneous*, *slope* **or** *derivative*, **nor the phrase** *rate of change*. **Do NOT use negative numbers in your answer.**

AT AN ELEVATION OF 3000 FEET, THE TEMPERATURE DROPS 4°C PER 1000 FEET INCREASE IN ELEVATION

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ on the same axes. **SCORE: ___ / 10 POINTS**



Consider the following 3 statements: **SCORE: ___ / 14 POINTS**

Statement 1: Rolle's Theorem applies to $f(x) = x^2 + 4x - 3$ on the interval $[0, 2]$

Statement 2: The Mean Value Theorem applies to $f(x) = |x|$ on the interval $[-3, 3]$

Statement 3: The Mean Value Theorem applies to $f(x) = \dfrac{1}{x}$ on the interval $[1, 5]$

[i] Determine which of the three statements above is true, and give a brief explanation why it is true.
NOTE: Only one statement is true.

#3: f IS CONT. ON $[1,5]$ AND DIFF ON $(1,5)$

[ii] Find the value of c guaranteed in the true statement in part [i].

$$f'(c) = \frac{f(5) - f(1)}{5 - 1}$$

$$-\frac{1}{c^2} = \frac{\frac{1}{5} - 1}{5 - 1} = \frac{-\frac{4}{5}}{4} = -\frac{1}{5}$$

$$c^2 = 5$$

$$c = \pm\sqrt{5} \qquad c = \sqrt{5} \in (1,5)$$

If $f(x)=\dfrac{1}{x^2-1}$, find $f'(2)$ **using the definition of the derivative at a point**. **SCORE: ___ / 10 POINTS**

SEE KEY +2t

Determine which of the following 3 statements is true. Show all relevant work. **SCORE: ___ / 8 POINTS**
NOTE: Only one statement is true.

Statement 1: $y=\ln(1-x^3)$ is orthogonal to $y=-\dfrac{1}{3x}-\dfrac{x^2}{6}$

Statement 2: $y=\ln(1-x^3)$ is orthogonal to $y=-\dfrac{3}{x}+\dfrac{x^2}{6}$

Statement 3: $y=\ln(1-x^3)$ is orthogonal to $y=-\dfrac{3}{x^2}-\dfrac{x^2}{6}$

#1: $y=\ln(1-x^3)$

$\dfrac{dy}{dx}=\dfrac{1}{1-x^3}\cdot -3x^2$

$=\dfrac{3x^2}{x^3-1}$

$y=-\dfrac{1}{3x}-\dfrac{x^2}{6}$

$\dfrac{dy}{dx}=\dfrac{1}{3x^2}-\dfrac{2x}{6}$

$=\dfrac{1}{3x^2}-\dfrac{x}{3}$

$=\dfrac{1-x^3}{3x^2}$

$\dfrac{3x^2}{x^3-1}\cdot\dfrac{1-x^3}{3x^2}=-1$

Evaluate the following derivatives. **Simplify your anwers (unless otherwise stated).** **SCORE: ___ / 32 POINTS**

[a] Find $\frac{d^2y}{dt^2}$ if $y = \frac{4-5t-6t^3}{\sqrt[3]{t}}$.

[b] Find $g'(x)$ if $g(x) = \sin^{-1}\left(x^2 - [f(x)]^3\right)$ where $f(x)$ is an unspecified function. **DO NOT SIMPLIFY YOUR ANSWER.**

[c] Find $\frac{dy}{dx}$ if $\tan(2x-3y) - e^{-x^2} = xy^3$.

SEE KEY +2t

[d] Find $f'(x)$ if $f(x) = \frac{\cos x^3}{\sin^2 x}$.

[e] Find $f'(x)$ if $f(x) = (\csc x)^{\cot x}$.