

SCORE: ____ / 100 POINTS

What month is your birthday ? ____
What are the first 2 digits of your address ? ____
What are the last 2 digits of your zip code ? ____
What are the last 2 digits of your social security number ? ____
**[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]**

CALCULATORS NOT ALLOWED

MULTIPLE CHOICE: The position of an object, in feet, at time t minutes is given by $s(t) = t^3 - 3t + 5$. SCORE: ____ / 5 POINTS
Find the average velocity of the object from time $t = 1$ to $t = 3$.

- [a] 10 ft/min [b] 11 ft/min [c] 15 ft/min [d] 16 ft/min

LETTER OF CORRECT ANSWER: [A] $\frac{s(3)-s(1)}{3-1}$

MULTIPLE CHOICE: The table below lists various values of $p(x)$ and $p'(x)$. SCORE: ____ / 5 POINTS
If $s(x) = \tan^{-1}(x + p(x))$, find the slope of the tangent line to $y = s(x)$ at $x = 1$.

x	-2	-1	0	1	2
$p(x)$	-1	2	-1	-3	1
$p'(x)$	3	-4	-2	3	5

- [a] -1 [b] 0.5 [c] 0.6 [d] 0.8

LETTER OF CORRECT ANSWER: [D] $\frac{1}{1+(x+p(x))^2} (1+p'(x))$

State the definition of instantaneous velocity. SCORE: ____ / 5 POINTS

SEE KEY + 2t

State Rolle's Theorem. SCORE: ____ / 5 POINTS

SEE KEY + 2t

Temperature tends to be lower at higher elevations (heights) than at lower elevations. If $T(x)$ is the temperature SCORE: ____ / 6 POINTS
(in degrees Celsius) at an elevation of x thousand feet, interpret the statement $T'(3) = -4$. Specify the units of all relevant numbers.
Do NOT use the words instantaneous, slope or derivative, nor the phrase rate of change. Do NOT use negative numbers in your answer.

SEE KEY -2t

Evaluate the following derivatives. Simplify your answers (unless otherwise stated).

SCORE: ____ / 32 POINTS

[a] Find $\frac{d^2 y}{dt^2}$ if $y = \frac{4 - 5t - 6t^3}{\sqrt[3]{t}}$.

[b] Find $g'(x)$ if $g(x) = \sin^{-1}(x^2 - [f(x)]^3)$ where $f(x)$ is an unspecified function. DO NOT SIMPLIFY YOUR ANSWER.

[c] Find $\frac{dy}{dx}$ if $\tan(2x - 3y) - e^{-x^2} = xy^3$.

SEE
KEY
+2t

[d] Find $f'(x)$ if $f(x) = \frac{\cos x^3}{\sin^2 x}$.

[e] Find $f'(x)$ if $f(x) = (\csc x)^{\cot x}$.

If $f(x) = \frac{1}{x^2 - 1}$, find $f'(2)$ using the definition of the derivative at a point.

SCORE: ____ / 10 POINTS

SEE KEY +2t

Determine which of the following 3 statements is true. Show all relevant work.

SCORE: ____ / 8 POINTS

NOTE: Only one statement is true.

Statement 1: $y = \ln(1 - x^3)$ is orthogonal to $y = -\frac{1}{3x} - \frac{x^2}{6}$

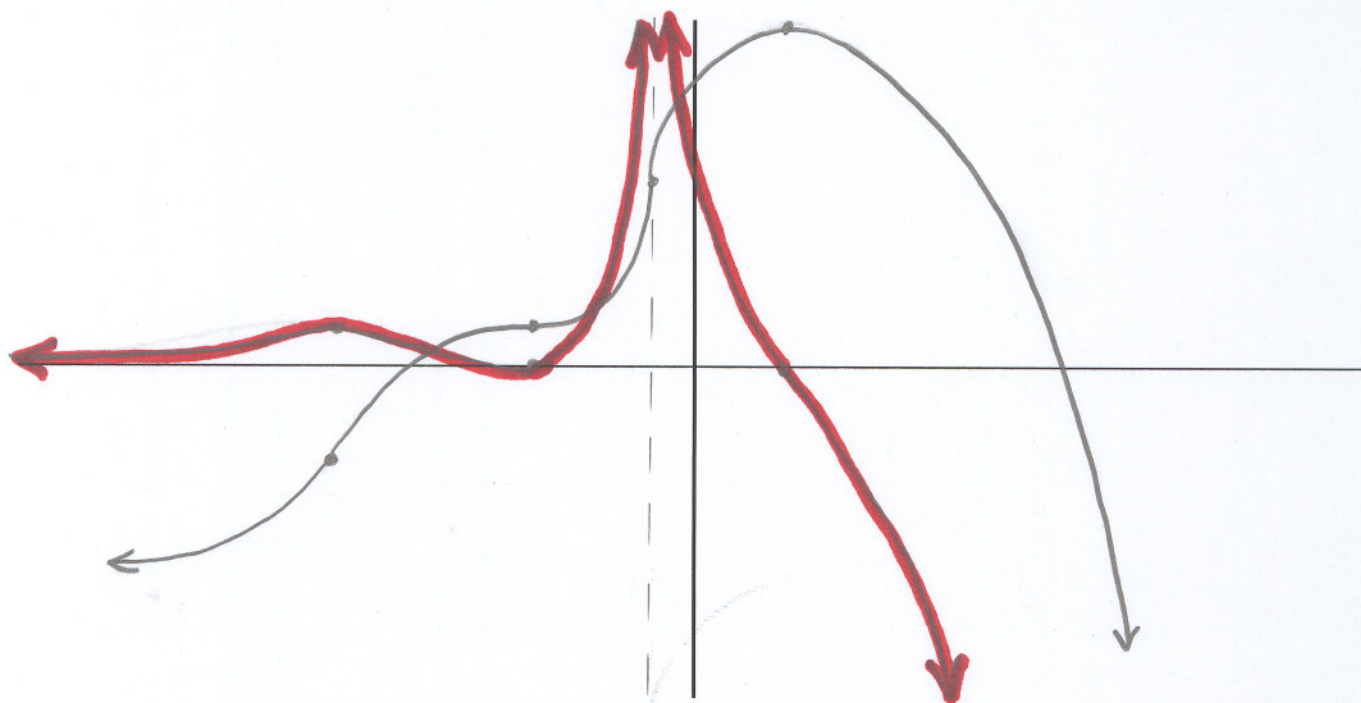
Statement 2: $y = \ln(1 - x^3)$ is orthogonal to $y = -\frac{3}{x} + \frac{x^2}{6}$

Statement 3: $y = \ln(1 - x^3)$ is orthogonal to $y = -\frac{3}{x^2} - \frac{x^2}{6}$

SEE KEY -2t

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ on the same axes.

SCORE: ___ / 10 POINTS



Consider the following 3 statements:

SCORE: ___ / 14 POINTS

Statement 1: Rolle's Theorem applies to $f(x) = x^2 + 4x - 3$ on the interval $[0, 5]$

Statement 2: The Mean Value Theorem applies to $f(x) = |x|$ on the interval $[-5, 5]$

Statement 3: The Mean Value Theorem applies to $f(x) = \frac{1}{x}$ on the interval $[1, 6]$

[i] Determine which of the three statements above is true, and give a brief explanation why it is true.

NOTE: Only one statement is true.

#3: f IS CONT. ON $[1, 6]$ AND DIFF. ON $(1, 6)$

[ii] Find the value of c guaranteed in the true statement in part [i].

$$f'(c) = \frac{f(6) - f(1)}{6 - 1}$$

$$-\frac{1}{c^2} = \frac{\frac{1}{6} - 1}{6 - 1} = \frac{-\frac{5}{6}}{5} = -\frac{1}{6}$$

$$c^2 = 6$$

$$c = \pm\sqrt{6}$$

$$c = \sqrt{6} \in (1, 6)$$