

What month is your birthday ?

What are the first 2 digits of your address ?

What are the last 2 digits of your zip code ?

What are the last 2 digits of your social security number ?

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]**NO CALCULATORS ALLOWED**

State the definition of the derivative of a function at a point.

SCORE: ___ / 2 POINTS

THE DERIVATIVE OF f AT $x=a$ IS

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$
Find the derivative of $f(x) = \frac{2x}{1-x}$ using the definition of the derivative.

SCORE: ___ / 5 POINTS

NOTE: You will receive 0 points if you use the differentiation shortcuts (power rule, product rule, quotient rule, chain rule etc.)

$$\begin{aligned}
 f'(x) &= \left[\lim_{h \rightarrow 0} \frac{\frac{2(x+h)}{1-(x+h)} - \frac{2x}{1-x}}{h} \right] 2 \\
 &= \lim_{h \rightarrow 0} \frac{2(x+h)(1-x) - 2x(1-x-h)}{h(1-x-h)(1-x)} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{2x} - \cancel{2x^2} + 2h - \cancel{2hx} - \cancel{2x} + \cancel{2x^2} + 2xh}{h(1-x-h)(1-x)} \\
 &= \left[\lim_{h \rightarrow 0} \frac{2}{(1-x-h)(1-x)} \right] = \left[\frac{2}{(1-x)^2} \right] 1
 \end{aligned}$$

The limit $\lim_{h \rightarrow 0} \frac{(h-1)^2 - 1}{h}$ equals $f'(a)$ for some function $f(x)$ and some constant a . Find $f(x)$ and a .

SCORE: ___ / 2 POINTS

$$\begin{aligned}
 f(x) &= x^2 \\
 a &= -1
 \end{aligned}$$

If $g(x) = x^3 f(x)$ and $f(-1) = 3$ and $f'(-1) = 5$, find $g'(-1)$.

SCORE: ___ / 2 POINTS

$$\begin{aligned}
 g'(x) &= 3x^2 f(x) + x^3 f'(x) \\
 g'(-1) &= 3(-1)^2 f(-1) + (-1)^3 f'(-1) = 3(3) - 5 = 4
 \end{aligned}$$

CONTINUED ON OTHER SIDE

[MULTIPLE CHOICE] If the position of an object at time t is given by $s(t) = (t^3 - 4t)\sqrt{t}$,
find the acceleration of the object at time $t = 1$.

SCORE: ___ / 2 POINTS

$$\uparrow a(1) = \frac{23}{4}$$

[a] $\frac{23}{4}$

[b] $\frac{13}{2}$

[c] $\frac{41}{4}$

[d] $\frac{47}{4}$

LETTER OF CORRECT ANSWER: []

$$\text{IF } s(t) = (t^3 + 4t)\sqrt{t}, a(1) = \frac{47}{4}$$

Find the derivatives of the following functions. Do **NOT** simplify your answers.

SCORE: ___ / 7 POINTS

[a] $r(x) = \frac{3x^4 + 2x^3 - 5}{5x^2 - 2x - 4}$

[b] $w(x) = \left(\frac{2}{x^2} - 1\right)(\sqrt[3]{x} + 3)\left(5 - \frac{1}{4\sqrt{x^7}}\right)$

$$r'(x) = \frac{(12x^3 + 6x^2)(5x^2 - 2x - 4) - (3x^4 + 2x^3 - 5)(10x - 2)}{(5x^2 - 2x - 4)^2}$$

+ $\frac{1}{2}$ POINT FOR CORRECT
(ORDER OF) SUBTRACTION

+ $\frac{1}{2}$ POINT FOR DIVISION

[YOU MAY USE THE SHORTCUT YOU FOUND IN
YOUR HOMEWORK]

$$w(x) = (2x^{-2} - 1)(x^{\frac{1}{3}} + 3)\left(5 - \frac{1}{4}x^{-\frac{7}{2}}\right)$$

$$w'(x) = (-4x^{-3})(x^{\frac{1}{3}} + 3)\left(5 - \frac{1}{4}x^{-\frac{7}{2}}\right) + (2x^{-2} - 1)\left(\frac{1}{3}x^{-\frac{2}{3}}\right)\left(5 - \frac{1}{4}x^{-\frac{7}{2}}\right) + (2x^{-2} - 1)(x^{\frac{1}{3}} + 3)\left(\frac{7}{8}x^{-\frac{9}{2}}\right)$$

+ $\frac{1}{2}$ POINT FOR ADDING
3 PRODUCTS TOGETHER

[1 BONUS POINT] Simplify $r'(x)$ in part [a] of the previous question.

$$r'(x) = \frac{60x^5 + 30x^4 - 24x^4 - 12x^3 - 48x^3 - 24x^2 - (30x^5 - 6x^4 + 20x^4 - 4x^3 - 50x + 10)}{(5x^2 - 2x - 4)^2}$$

$$= \frac{30x^5 - 8x^4 - 56x^3 - 24x^2 + 50x - 10}{(5x^2 - 2x - 4)^2}$$

NO PARTIAL
CREDIT