

What month is your birthday ?

What are the first 2 digits of your address ?

What are the last 2 digits of your zip code ?

What are the last 2 digits of your social security number ?

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]**NO CALCULATORS ALLOWED**

State the definition of the derivative of a function at a point.

SCORE: ___ / 2 POINTS

THE DERIVATIVE OF f AT $x=a$ IS

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Find the derivative of $f(x) = \frac{x}{1-2x}$ using the definition of the derivative.

SCORE: ___ / 5 POINTS

NOTE: You will receive 0 points if you use the differentiation shortcuts (power rule, product rule, quotient rule, chain rule etc.)

$$f'(x) = \left[\lim_{h \rightarrow 0} \frac{\frac{x+h}{1-2(x+h)} - \frac{x}{1-2x}}{h} \right] \quad 2 \text{ POINTS}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)(1-2x) - x(1-2x-2h)}{h(1-2x-2h)(1-2x)}$$

$$= \lim_{h \rightarrow 0} \frac{x - 2x^2 + h - 2xh - x + 2x^2 + 2xh}{h(1-2x-2h)(1-2x)}$$

$$2 \text{ POINTS} \quad = \left[\lim_{h \rightarrow 0} \frac{1}{(1-2x-2h)(1-2x)} \right] = \left[\frac{1}{(1-2x)^2} \right] \quad 1 \text{ POINT}$$

[MULTIPLE CHOICE] If the position of an object at time t is given by $s(t) = (t^3 - 4t)\sqrt{t}$,
find the acceleration of the object at time $t = 1$.

SCORE: ___ / 2 POINTS

$$\uparrow a(1) = \frac{23}{4}$$

[a] $\frac{13}{2}$

[b] $\frac{47}{4}$

[c] $\frac{23}{4}$

[d] $\frac{41}{4}$

LETTER OF CORRECT ANSWER: []

$$\text{IF } s(t) = (t^3 + 4t)\sqrt{t}, \quad a(1) = \frac{47}{4}$$

The limit $\lim_{h \rightarrow 0} \frac{(h-2)^4 - 16}{h}$ equals $f'(a)$ for some function $f(x)$ and some constant a . Find $f(x)$ and a . SCORE: ___ / 2 POINTS

$$f(x) = x^4 \quad] \quad 1 \text{ POINT}$$

$$a = -2 \quad] \quad 1 \text{ POINT}$$

CONTINUED ON OTHER SIDE

If $g(x) = x^4 f(x)$ and $f(-1) = 3$ and $f'(-1) = 5$, find $g'(-1)$.

SCORE: ___ / 2 POINTS

$$g'(x) = 4x^3 f(x) + x^4 f'(x) \quad] \text{ 1 POINT}$$

$$g'(-1) = 4(-1)^3 f(-1) + (-1)^4 f'(-1) = -4(3) + 5 = -7$$

1 POINT

Find the derivatives of the following functions. Do **NOT** simplify your answers.

SCORE: ___ / 7 POINTS

[a] $w(x) = \left(\frac{2}{x^2} - 1\right)(\sqrt[3]{x} + 3)\left(5 - \frac{1}{4\sqrt{x^7}}\right)$

[b] $r(x) = \frac{3x^4 + 2x^3 - 5}{5x^2 - 2x - 4}$

[YOU MAY USE THE SHORTCUT YOU FOUND IN YOUR HOMEWORK]

$$w(x) = (2x^{-2} - 1)(x^{\frac{1}{3}} + 3)(5 - \frac{1}{4}x^{-\frac{7}{2}})$$

$$w'(x) = \underbrace{(-4x^{-3})(x^{\frac{1}{3}} + 3)(5 - \frac{1}{4}x^{-\frac{7}{2}})}_{\text{1}} + \underbrace{(2x^{-2} - 1)(\frac{1}{3}x^{-\frac{2}{3}})(5 - \frac{1}{4}x^{-\frac{7}{2}})}_{\text{2}} + \underbrace{(2x^{-2} - 1)(x^{\frac{1}{3}} + 3)(\frac{7}{8}x^{-\frac{9}{2}})}_{\text{3}}$$

+ 1/2 POINT FOR ADDING THE 3 PRODUCTS TOGETHER

$$r'(x) =$$

$$\frac{[(12x^3 + 6x^2)(5x^2 - 2x - 4)]' - (3x^4 + 2x^3 - 5)(10x - 2)'}{(5x^2 - 2x - 4)^2} \quad \frac{1}{2}$$

+ 1/2 POINT FOR CORRECT (ORDER OF) SUBTRACTION
+ 1/2 POINT FOR DIVISION

[1 BONUS POINT] Simplify $r'(x)$ in part [b] of the previous question.

$$r'(x) = \frac{[60x^5 + 30x^4 - 24x^4 - 12x^3 - 48x^3 - 24x^2 - (30x^5 - 6x^4 + 20x^4 - 4x^3 - 50x + 10)]}{(5x^2 - 2x - 4)^2}$$

$$= \frac{30x^5 - 8x^4 - 56x^3 - 24x^2 + 50x - 10}{(5x^2 - 2x - 4)^2} \quad]$$

NO PARTIAL CREDIT