

NO CALCULATORS ALLOWED

[MULTIPLE CHOICE] If the position of an object at time t is given by $s(t) = e^{-at} \cos bt$,
find the acceleration of the object at time $t = 0$.

SCORE: ____ / 2 POINTS

Since acceleration is the 2nd derivative of position with respect to time,

and $s''(t) = (a^2 - b^2)e^{-at} \cos bt + 2abe^{-at} \sin bt$, therefore the answer is $a^2 - b^2$

Find the following derivatives. Simplify by factoring where appropriate.

SCORE: ____ / 18 POINTS

$\frac{1}{2}$ point for each item marked in yellow

$\frac{1}{4}$ point for each item marked in grey

1 point for each simplified answer

[a] If $f(x) = 3^{x^2} \sec^2 x$, find $f'(x)$.

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{2}$

$$f'(x) = 3^{x^2} (\ln 3)(2x) \sec^2 x + 3^{x^2} (2 \sec x)(\sec x \tan x)$$

$$f'(x) = (2 \cdot 3^{x^2} \sec^2 x)[(\ln 3)x + \tan x]$$

↑ 1 point for final simplification

TOTAL FOR THIS PART: 4½ POINTS

[b] If $f(x) = \cot^2(\cos x)$, find $f'(x)$.

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$$f'(x) = 2 \cot(\cos x)(-\csc^2(\cos x))(-\sin x)$$

$$f'(x) = 2 \cot(\cos x) \csc^2(\cos x) \sin x$$

↑ 1 point for final simplification

TOTAL FOR THIS PART: 2½ POINTS

[c] If $f(x) = \frac{e^{\sin x}}{3 - \tan x}$, find $f'(x)$.

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{2}$ for denominator

$$f'(x) = \frac{e^{\sin x}(\cos x)(3 - \tan x) - e^{\sin x}(-\sec^2 x)}{(3 - \tan x)^2}$$

$$f'(x) = \frac{e^{\sin x}(3 \cos x - \sin x + \sec^2 x)}{(3 - \tan x)^2}$$

↑ 1 point for final simplification

TOTAL FOR THIS PART: 3¾ POINTS

[d] If $f(x) = x^3 + 2x$, and $g(x) = f^{-1}(x)$, find $g'(12)$.

$\frac{1}{2}$ $\frac{1}{2}$

$$f(2) = 12 \quad \text{so} \quad g'(12) = \frac{1}{f'(2)}$$

$$f'(x) = 3x^2 + 2 \quad \text{so} \quad g'(12) = \frac{1}{3(2)^2 + 2} = \frac{1}{14}$$

↑
 $\frac{1}{2}$

↑
 $\frac{1}{2}$

TOTAL FOR THIS PART: 2 POINTS

[e] If $f(x) = x^2 e^{-x} \csc 3x$, find $f'(x)$.

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{2}$

$$f'(x) = 2xe^{-x} \csc 3x + x^2 e^{-x}(-1) \csc 3x + x^2 e^{-x}(-\csc 3x \cot 3x)(3)$$

$$f'(x) = (xe^{-x} \csc 3x)(2 - x - 3x \cot 3x)$$

↑ 1 point for final simplification

TOTAL FOR THIS PART: 5½ POINTS