Math 1A (9:30am – 10:20am)
Quiz 6
Fri Feb 20, 2009 **DUE MON FEB 23, 2009 IN CLASS**

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

Show that the families of curves $x = ky^5$ and $5x^2 + y^2 = c$ are orthogonal for all constants k and c. **SCORE: ___ / 4 POINTS**

$1 = 5ky^4 \frac{dy}{dx}$ $\qquad$ $10x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{1}{5ky^4}$ 1 $\qquad$ $\frac{dy}{dx} = \frac{-5x}{y}$ 1

$\frac{1}{5ky^4} \cdot \frac{-5x}{y}$ $\frac{1}{2}$

$= \frac{-x}{ky^5}$

$= \frac{-x}{x}$ 1

$= -1$ $\frac{1}{2}$

$\boxed{\text{OR}}$ $\frac{1}{5ky^4} = \frac{1}{5\left(\frac{x}{y^5}\right)y^4}$

$= \frac{1}{\frac{5x}{y}} = \frac{y}{5x}$ 1

$\frac{-5x}{y} \cdot \frac{y}{5x} = -1$ $\frac{1}{2}$ $\frac{1}{2}$

Determine if each statement below is true or false (circle TRUE or FALSE), and give a **brief** explanation. You will receive 0 credit if you circle TRUE or FALSE but you do not give an explanation. **SCORE: ___ / 8 POINTS**

TRUE FALSE The Mean Value Theorem applies to the function $f(x) = \sqrt[3]{x}$ on the interval $[-1, 1]$. 1

f NOT DIFF AT $x = 0$ 1

SINCE $f' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

TRUE FALSE Rolle's Theorem applies to the function $f(x) = \frac{1}{x^2}$ on the interval $[-2, 2]$. 1

f NOT CONT AT $x = 0$ 1

TRUE FALSE Rolle's Theorem applies to the function $f(x) = x^3$ on the interval $[-2, 2]$. 1

$f(-2) = -8$

$f(2) = 8$

$f(-2) \neq f(2)$ 1

TRUE FALSE The Mean Value Theorem applies to the function $f(x) = \tan x$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. 1

f IS CONT. ON $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ $\frac{1}{2}$

AND DIFF. ON $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ $\frac{1}{2}$

CONTINUED ON OTHER SIDE

Find the derivative of $f(x) = (\sec x)^{\cos x}$. Simplify your answer. SCORE: ___ / 3 POINTS

$\frac{1}{2}$ $\ln f(x) = \cos x \ln \sec x$

$\frac{1}{2}$ $\frac{1}{f(x)} f'(x) = -\sin x \ln \sec x + \cos x \frac{1}{\cancel{\sec x}} \cancel{\sec x} \tan x$ 1

$= -\sin x \ln \sec x + \sin x$ $\frac{1}{2}$

$f'(x) = f(x) \sin x (1 - \ln \sec x)$

$= \sin x (1 - \ln \sec x)(\sec x)^{\cos x}$ $\frac{1}{2}$

OK IF NOT FACTORED

Prove that $f(x) = x^4 + 4x^2 - 4$ has exactly 2 zeros. SCORE: ___ / 5 POINTS

<u>You must provide a logical and fully justified argument to receive full credit.</u>

SINCE f IS A POLYNOMIAL,

THEREFORE f IS CONTINUOUS AND DIFFERENTIABLE EVERYWHERE

SINCE $f(-1) = 1$ AND $f(0) = -4$ AND $f(1) = 1$

SO $f(0) < 0 < f(-1)$ AND $f(0) < 0 < f(1)$

SO BY IVT, THERE ARE AT LEAST 2 ZEROS,

ONE IN $(-1, 0)$ AND ONE IN $(0, 1)$

SUPPOSE f HAS AT LEAST 3 ZEROS,

THEN BY TH'M 9.3, f' HAS AT LEAST 2 ZEROS,

BUT $f'(x) = 4x^3 + 8x = 4x(x^2+2) = 0$ ONLY WHEN $x = 0$

SO f DOES NOT HAVE AT 3 ZEROS,

SO f MUST HAVE EXACTLY 2 ZEROS

$\frac{1}{2}$ POINT EACH