Math 1A (7:30am – 8:20am)
Midterm 1 Version A
Tue Oct 12, 2010

NAME YOU ASKED TO BE CALLED: ____________________

SCORE: ___ / 150 POINTS

NO CALCULATORS OR DIFFERENTIATION SHORTCUTS (FROM CH 3) ALLOWED
SHOW PROPER CALCULUS-LEVEL ALGEBRAIC WORK AND USE PROPER NOTATION

Give the definition of "horizontal asymptote". **SCORE: ___ / 6 POINTS**

SEE QUIZ 3 SOLUTION

Let $f(x) = \begin{cases} x^{-1} & \text{if } x < 1 \\ x^2 - 2 & \text{if } 1 < x < 4 \\ 3x + 2 & \text{if } x > 4 \end{cases}$. **SCORE: ___ / 20 POINTS**

Find all discontinuities of $f(x)$ and classify each as removable, jump or infinite. Justify your answer algebraically, without using a graph.

f IS DISCONT AT $x = 0, 1, 4$ SINCE $f(0), f(1), f(4)$ DNE

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \frac{1}{x} = \infty$ SO $x = 0$ IS AN INFINITE DISCONT

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} x^{-1} = 1$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (x^2 - 2) = -1$

$\lim\limits_{x \to 1^-} f(x) \neq \lim\limits_{x \to 1^+} f(x)$

SO $x = 1$ IS A JUMP DISCONT

$\lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^-} (x^2 - 2) = 14$

$\lim\limits_{x \to 4^+} f(x) = \lim\limits_{x \to 4^+} (3x + 2) = 14$

$\lim\limits_{x \to 4} f(x) = 14$ BUT $f(4)$ DNE

SO $x = 4$ IS A REMOVABLE DISCONT

Find the equations of the vertical asymptotes of $f(x) = \dfrac{1 - x}{x^3 - 4x^2 + 4x}$. **SCORE: ___ / 18 POINTS**

Find the values of both one-sided limits at each vertical asymptote.
When showing your "work", you may use the shorthand notation shown in class.

$f(x) = \dfrac{1 - x}{x(x - 2)^2}$

f HAS V.A. @ $x = 0, 2$

$\lim\limits_{x \to 0^-} \dfrac{1 - x}{x(x-2)^2} = -\infty$

$\dfrac{1}{0^- \cdot 4}$

$\lim\limits_{x \to 2^-} \dfrac{1 - x}{x(x-2)^2} = -\infty$

$\dfrac{-1}{2 \cdot 0^+}$

$\lim\limits_{x \to 0^+} \dfrac{1 - x}{x(x-2)^2} = \infty$

$\dfrac{1}{0^+ \cdot 4}$

$\lim\limits_{x \to 2^+} \dfrac{1 - x}{x(x-2)^2} = -\infty$

$\dfrac{-1}{2 \cdot 0^+}$

Find the equations of the horizontal asymptotes of $f(x) = \dfrac{7-3e^x}{\sqrt{4e^{2x}+6}}$.

SCORE: ___ / 15 POINTS

$$\lim_{x\to-\infty} \frac{7-3e^x}{\sqrt{4e^{2x}+6}} = \frac{7-3(0)}{\sqrt{4(0)+6}} = \frac{7}{\sqrt{6}} \qquad y = \frac{7}{\sqrt{6}}$$

$$\lim_{x\to\infty} \frac{7-3e^x}{\sqrt{4e^{2x}+6}} = \lim_{x\to\infty} \frac{7-3e^x}{\sqrt{4e^{2x}+6}} \cdot \frac{e^{-x}}{e^{-x}}$$

$\frac{-\infty}{\infty}$

$$= \lim_{x\to\infty} \frac{7e^{-x}-3}{\sqrt{4+6e^{-2x}}}$$

$$= \frac{0-3}{\sqrt{4+0}}$$

$$= \frac{-3}{2} \qquad y = -\frac{3}{2}$$

The position of an object (in feet) at time t minutes, is given by the function $s(t) = \dfrac{t}{\sqrt{t+6}}$.

SCORE: ___ / 18 POINTS

Find the instantaneous velocity of the object at time $t = 3$. Specify the units.

$$\lim_{b\to3} \frac{\frac{b}{\sqrt{b+6}} - 1}{b-3}$$

$$= \lim_{b\to3} \frac{b-\sqrt{b+6}}{(b-3)\sqrt{b+6}}$$

$$= \lim_{b\to3} \frac{b^2-(b+6)}{(b-3)\sqrt{b+6}\,(b+\sqrt{b+6})}$$

$$= \lim_{b\to3} \frac{\cancel{(b-3)}(b+2)}{\cancel{(b-3)}\sqrt{b+6}\,(b+\sqrt{b+6})}$$

$$= \frac{5}{3(6)}$$

$$= \frac{5}{18}$$

Suppose you are given a graph of $y = 2x^3 - 5x^2 - 1$. **SCORE: ___ / 18 POINTS**

[a] Find $\dfrac{dy}{dx}$.

$$\lim_{h \to 0} \frac{2(x+h)^3 - 5(x+h)^2 - 1 - 2x^3 + 5x^2 + 1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 - \cancel{5x^2} - 10xh - 5h^2 - \cancel{1} - \cancel{2x^3} + \cancel{5x^2} + \cancel{1}}{h}$$

$$= \lim_{h \to 0} (6x^2 + 6xh + 2h^2 - 10x - 5h)$$

$$= 6x^2 - 10x$$

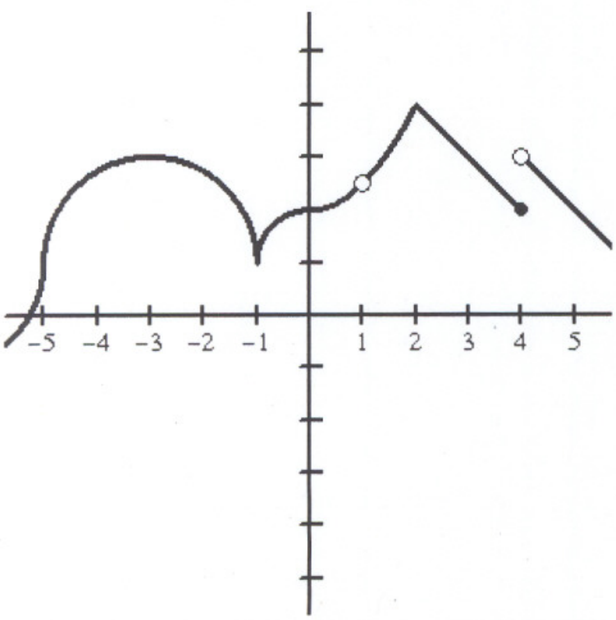
[b] Find the equation of the tangent line to the graph above at $x = 2$.

$$\left.\frac{dy}{dx}\right|_{x=2} = 24 - 20 = 4$$

WHEN $x = 2$, $y = 16 - 20 - 1 = -5$

$$y + 5 = 4(x - 2)$$

The graph of $f(x)$ is shown below. Find all x-coordinates where $f'(x)$ is undefined, and explain briefly why. **SCORE: ___ / 10 POINTS**



DISCONT @ $x = 1, 4$

CUSP @ $x = -1, 2$

VERTICAL T.L. @ $x = -5$ (AND -1)

STUDENT'S CHOICE: Circle the question you want to be graded
If no question is circled, only Choice #1 will be graded

CHOICE #1:

SCORE: ___ / 12 POINTS

State both the Squeeze Theorem and the Intermediate Value Theorem.

SEE QUIZ 3 SOLUTIONS

CHOICE #2:

SCORE: ___ / 12 POINTS

One of the three statements below can be proven using the Intermediate Value Theorem (IVT).
Circle the statement that can be proven using the IVT, and write the proof.

Statement #1: $f(x) = \dfrac{x^4 - x - 1}{x^2 - 9}$ has a zero in the interval $[-2, 4]$.

Statement #2: $f(x) = \dfrac{x^4 - x - 1}{x^2 - 9}$ has a zero in the interval $[-2, 2]$.

Statement #3: $f(x) = \dfrac{x^4 - x - 1}{x^2 - 9}$ has a zero in the interval $[-2, 0]$.

$\dfrac{x^4 - x - 1}{x^2 - 9}$ IS DISCONT @ $x = \pm 3$

$f(-2) < 0$ $f(4) > 0$

$f(2) < 0$

$f(0) > 0$

$f(x) = \dfrac{x^4 - x - 1}{x^2 - 9}$ IS CONT. ON $[-2, 0]$

SINCE f IS RATIONAL AND ITS DOMAIN IS $\{x \neq \pm 3\}$

$f(-2) = \dfrac{16 - {}^{-}2 - 1}{4 - 9} = -\dfrac{17}{5}$ $f(0) = \dfrac{1}{9}$

$f(-2) < 0 < f(0)$

BY IVT, THERE IS A $c \in [-2, 0]$ SUCH THAT $f(c) = 0$.

The number of pictures you can store on your computer depends on the size of each picture. **SCORE: ___ / 15 POINTS**

Let $p = g(s)$, where p is the number of pictures (in hundreds of pictures), and s is the size of each picture (in kilobytes).

[a] What are the units of $g'(s)$? **DO NOT SIMPLIFY.**

$$\frac{\text{UNITS OF } p}{\text{UNITS OF } s} = \frac{\text{HUNDREDS OF PICTURES}}{\text{KILOBYTE}}$$

[b] Give the practical meaning (including units) of $g'(12) = -9$.

IF EACH PICTURE IS 12 KILOBYTES,
THE COMPUTER WILL HOLD 900 FEWER PICTURES
FOR EACH 1 KILOBYTE INCREASE IN PICTURE SIZE

[c] Is there a value of s_0 for which you would expect $g'(s_0) > 0$? Why or why not ?

NO. AS THE PICTURES GET LARGER, THE COMPUTER
WILL HOLD FEWER + FEWER OF THEM.

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ on the same axes. **SCORE: ___ / 18 POINTS**