Math 1A Midterm 1 Review

1. THERE WILL BE VERY FEW PROBLEMS THAT EXPLICITLY ASK YOU TO FIND THE LIMIT OF A FUNCTION. INSTEAD, YOU WILL BE ASKED TO SOLVE PROBLEMS WHERE YOU NEED TO WRITE YOUR OWN LIMITS, THEN FIND THEM.
2. IF YOU HAVE TAKEN DIFFERENTIAL CALCULUS BEFORE, DO NOT USE DIFFERENTIATION SHORTCUTS.
3. YOU SHOULD ONLY REQUIRE A CALCULATOR FOR QUESTIONS MARKED [C].
4. UNLESS A GRAPH IS GIVEN, YOU MUST BE ABLE TO SOLVE EACH PROBLEM WITHOUT A GRAPH.

[1][C] Estimate the slope of the tangent line to the curve $y = \sqrt{x + \sqrt{\cos x}}$ at the point $(0, 1)$ using the slopes of several secant lines.

[2] The position of an object (in meters) at time t seconds, is given by the function $f(t) = t^2 \cos \pi t$. Find the average velocity of the object over the interval $[1, 5]$. Specify the units.

[3] Sketch the graph of a function $f(x)$ which satisfies the following conditions:

$\lim_{x \to -2^+} g(x) = -3$, $\lim_{x \to -2^-} g(x) = \infty$, $\lim_{x \to 1} g(x) = -\infty$, $\lim_{x \to -\infty} g(x) = 2$, and $\lim_{x \to \infty} g(x) = -2$

[4] Prove that $\lim_{x \to 0} x \cos \frac{1}{x^2} = 0$.

[5] Let $f(x) = \begin{cases} 2x - 3 & \text{if } x < -1 \\ x^2 - 6 & \text{if } -1 < x < 2 \\ 4x - 6 & \text{if } x \geq 2 \end{cases}$.

[a] Find $\lim_{x \to -2} f(x)$.

[b] Find $\lim_{x \to -1} f(x)$.

[c] Find $\lim_{x \to 2} f(x)$.

[6] Find the value of a if $\lim_{x \to 2} \frac{\sqrt{x^2 + a} - 1}{x - 2} = 2$.

[7] If $\lim_{x \to 2} f(x) = -3$ and $\lim_{x \to 2} g(x) = 4$, find $\lim_{x \to 2} \frac{x^2 g(x)}{1 + f(x)}$. Show clearly how the limit laws are used in your solution.

[8] Find the discontinuities of $f(x) = \frac{x + 2}{x^2 - 9}$, and find the one-sided limits at each discontinuity.

[9] Let $f(x) = \begin{cases} 2x + a & \text{if } x < -1 \\ 3 - x & \text{if } -1 < x < 2 \\ bx - 1 & \text{if } x \geq 2 \end{cases}$.

[a] Find the value of a so that $f(x)$ is continuous at $x = -1$.

[b] Find the value of b so that $f(x)$ is continuous at $x = 2$.

[c] If $a = 6$ and $b = 3$, find all discontinuities of $f(x)$ and find the type of each discontinuity (removable, jump or infinite).

[10] Use the Intermediate Value Theorem to prove that the equation $\cos 2x = x^2$ has a solution in the interval $[0, \pi]$.

[11] Find all horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{4 + 9x^2}}{2x - 1}$.

[12] If $f(x) = x^3 - 3x + 2$, find $f'(-2)$ using both definitions of $f'(a)$.

[13] Find a function f and a number a such that the derivative of f at a is given by

[a] $\lim\limits_{h \to 0} \dfrac{\cos(\pi(h-1))+1}{h}$ [b] $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x+2}$

[14] The position of an object (in feet) at time t minutes, is given by the function $f(t) = \sqrt{t^2 - 5}$. Find the instantaneous velocity of the object at time $t = 3$. Specify the units.

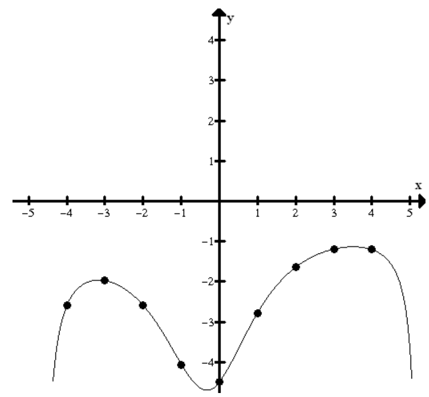
[15] Find the equation of the tangent line to the curve of $f(x) = \dfrac{2x}{1-x}$ at $x = 2$.

[16] The graph of f is shown to the right. Arrange the following from least (most negative) to greatest (most positive).

0 $f'(-4)$ $f'(-2)$ $f'(2)$ $f'(4)$

__________ < __________ < __________ < __________ < __________

LEAST GREATEST



[17] The time required to defrost a piece of frozen meat in the refrigerator depends on the temperature inside the refrigerator. Let $t = f(T)$, where t is the defrost time (in hours), and T is the refrigerator temperature (in $^\circ C$)

[a] Give the practical meaning (including units) of $f'(4) = 6$.

[b] Give the practical meaning (including units) of $f'(4) = -1$.

[c] Is there a value of T_0 for which you would expect $f'(T_0) > 0$? Why or why not ?

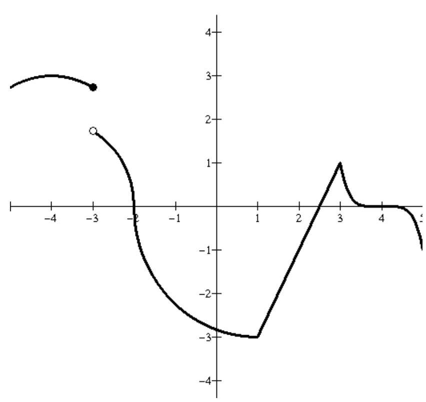
[18] Using the definition of the derivative, find the derivatives of the following functions.

[a] $f(t) = \dfrac{1}{\sqrt{1-t}}$ [b] $g(x) = \dfrac{4x}{2-x}$

[19] The graph of $f(x)$ is shown on the right.

[a] Find all x-coordinates where $f'(x)$ is undefined, and explain briefly why.

[b] Sketch a graph of $f'(x)$.



[20] If the tangent line to the graph of $y = f(x)$ at $x = 4$ is $x - 2y = 6$, prove that $\lim\limits_{x \to 4} f(x) = -1$.

YOU MUST ALSO KNOW THE FOLLOWING DEFINITIONS AND THEOREMS:

Definitions

- vertical/horizontal asymptote
- continuity at a point
- removable discontinuity (from lecture)
- jump discontinuity (from lecture)
- derivative at a point
- derivative function

Theorems

- Squeeze Theorem
- Intermediate Value Theorem
- Differentiability implies continuity