Math 1A (7:30am – 8:20am)
Midterm 2 Version A
Tue Nov 2, 2010

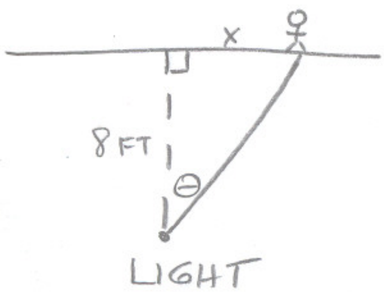
NAME YOU ASKED TO BE CALLED: ____________________

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

A man walks along a straight path at a speed of 5 feet per second. A searchlight is located on the ground 8 feet from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 6 feet from the point on the path closest to the searchlight? **Specify the units for your final answer.**

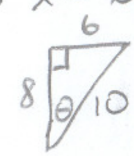
SCORE: _____ / 22 POINTS



$\frac{dx}{dt} = 5 \text{ ft/sec}$

WANT $\frac{d\theta}{dt}$ WHEN $x = 6 \text{ ft}$

$\tan\theta = \frac{x}{8 \text{ ft}}$ $\longrightarrow$ WHEN $x = 6 \text{ ft}$,



$\sec\theta = \frac{10}{8} = \frac{5}{4}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{8 \text{ ft}} \frac{dx}{dt}$

$\left(\frac{5}{4}\right)^2 \frac{d\theta}{dt} = \frac{1}{8 \cancel{\text{ft}}} \frac{5 \cancel{\text{ft}}}{\text{sec}}$

$\frac{d\theta}{dt} = \frac{5}{8}\left(\frac{16}{25}\right)/\text{sec} = \frac{2}{5} \text{ RADIANS/SEC}$

If $q(t) = (\csc t)^{\tan t}$, find $q'(t)$.

SCORE: _____ / 22 POINTS

$\ln q(t) = \tan t \ln \csc t$

$\frac{q'(t)}{q(t)} = \sec^2 t \ln \csc t + \cancel{\tan t} \cdot \frac{-\cancel{\csc t \cot t}}{\cancel{\csc t}}$

$q'(t) = q(t)\left[\sec^2 t \ln \csc t - 1\right]$

$= (\csc t)^{\tan t}(\sec^2 t \ln \csc t - 1)$

The position of an object at time t is given by $s(t) = \tan^{-1} t^3$. SCORE: _____ / 22 POINTS

Find the **acceleration** of the object at time $t = 1$.

$$s'(t) = \frac{3t^2}{1+(t^3)^2} = \frac{3t^2}{1+t^6}$$

$$s''(t) = \frac{6t(1+t^6) - 3t^2(6t^5)}{(1+t^6)^2}$$

$$s''(1) = \frac{6(1)(2) - 3(1)(6)}{2^2} = \frac{-6}{4} = -\frac{3}{2}$$

Find the equation of the normal line to $y = 3^{\arcsin x} + \sec x$ at $x = 0$. SCORE: _____ / 15 POINTS

$$y' = 3^{\arcsin x} \cdot \ln 3 \cdot \frac{1}{\sqrt{1-x^2}} + \sec x \tan x$$

$$y'(0) = 3^0 \cdot \ln 3 \cdot \frac{1}{\sqrt{1}} + 1 \cdot 0 = \ln 3$$

$$y(0) = 3^0 + 1 = 2$$

$$m = -\frac{1}{\ln 3}$$

$$y - 2 = -\frac{1}{\ln 3}(x - 0) \quad \text{OR} \quad y = 2 - \frac{x}{\ln 3}$$

Show that $x^2 + y^2 = ax$ and $x^2 + y^2 = by$ are orthogonal trajectories (where a and b are constants). SCORE: _____ / 22 POINTS

$$x^2 + y^2 = ax \qquad\qquad x^2 + y^2 = by$$

$$2x + 2y\frac{dy}{dx} = a \qquad\qquad 2x + 2y\frac{dy}{dx} = b\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{a-2x}{2y} \qquad\qquad \frac{dy}{dx} = \frac{2x}{b-2y}$$

$$\frac{a-2x}{\cancel{2}y} \cdot \frac{\cancel{2}x}{b-2y} = \frac{ax-2x^2}{by-2y^2} = \frac{x^2+y^2-2x^2}{x^2+y^2-2y^2} = \frac{y^2-x^2}{x^2-y^2}$$

$$= -1$$

If $g(x)=\dfrac{(3x+2)^2}{\sqrt[3]{x}}$, find $g''(x)$. **SIMPLIFY YOUR ANSWER, AND FACTOR.** **SCORE: _____ / 10 POINTS**

$$g(x) = (9x^2+12x+4)x^{-\frac{1}{3}} = 9x^{\frac{5}{3}} + 12x^{\frac{2}{3}} + 4x^{-\frac{1}{3}}$$

$$g'(x) = 15x^{\frac{2}{3}} + 8x^{-\frac{1}{3}} - \frac{4}{3}x^{-\frac{4}{3}}$$

$$g''(x) = 10x^{-\frac{1}{3}} - \frac{8}{3}x^{-\frac{4}{3}} + \frac{16}{9}x^{-\frac{7}{3}}$$

$$= \frac{2}{9}x^{-\frac{7}{3}}(45x^2 - 12x + 8)$$

State the definition of e from section 3.1 of the textbook (the definition used during the proof that $(e^x)' = e^x$.) **SCORE: _____ / 7 POINTS**

e IS THE NUMBER SUCH THAT $\lim\limits_{h\to 0} \dfrac{e^h - 1}{h} = 0$

Using the definition of the derivative, prove the derivative of $f(x) = \sec x$. **SCORE: _____ / 22 POINTS**

<u>You may use the two trigonometric limits proved in class, without reproving them. You MUST NOT use any differentiation shortcuts.</u>

$$\frac{d}{dx}\sec x = \lim_{h\to 0} \frac{\sec(x+h) - \sec x}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h}$$

$$= \lim_{h\to 0} \frac{\cos x - \cos(x+h)}{h\cos(x+h)\cos x}$$

$$= \lim_{h\to 0} \frac{\cos x - \cos x\cos h + \sin x \sin h}{h\cos(x+h)\cos x}$$

$$= \lim_{h\to 0} \frac{-\cos x\left(\frac{\cos h - 1}{h}\right) + \sin x\left(\frac{\sin h}{h}\right)}{\cos(x+h)\cos x}$$

$$= \frac{(-\cos x)(0) + (\sin x)(1)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \sec x \tan x$$

The amount of storage you need for your photo collection depends on the resolution of each photo. **SCORE: ___ / 8 POINTS**

If $s = f(r)$, where s is the storage needed (in megabytes), and r is the resolution of each photo (in hundreds of dots per inch), what does the statement $f'(9) = 11$ mean ? Give the units of measurement for each number in your answer.

<u>NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".</u>

IF THE RESOLUTION OF EACH PHOTO IS 900 DOTS PER INCH, YOU NEED AN ADDITIONAL 11 MEGABYTES FOR YOUR PHOTO COLLECTION FOR EACH 100 DOT PER INCH INCREASE IN EACH PHOTO'S RESOLUTION

BONUS POINTS ON OTHER SIDE ➡