Math 1A (7:30am – 8:20am)
Midterm 2 Version C
Tue Nov 2, 2010

NAME YOU ASKED TO BE CALLED: ____________________

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

The position of an object at time t is given by $s(t) = \tan^{-1} t^4$.
Find the **acceleration** of the object at time $t = 1$.

SCORE: _____ / 22 POINTS

$$s'(t) = \frac{4t^3}{1+(t^4)^2} = \frac{4t^3}{1+t^8}$$

$$s''(t) = \frac{12t^2(1+t^8) - 4t^3(8t^7)}{(1+t^8)^2}$$

$$s''(1) = \frac{12(1)(2) - 4(1)(8)}{2^2} = \frac{-8}{4} = -2$$

Using the definition of the derivative, prove the derivative of $f(x) = \sec x$.

SCORE: _____ / 22 POINTS

You may use the two trigonometric limits proved in class, without reproving them. You MUST NOT use any differentiation shortcuts.

SEE VERSION A KEY

Find the equation of the normal line to $y = 5^{\arcsin x} + \tan x$ at $x = 0$. **SCORE: _____ / 15 POINTS**

$y' = 5^{\arcsin x} \cdot \ln 5 \cdot \frac{1}{\sqrt{1-x^2}} + \sec^2 x$

$y'(0) = 5^0 \cdot \ln 5 \cdot \frac{1}{\sqrt{1}} + 1^2 = \ln 5 + 1$ or $1 + \ln 5$

$y(0) = 5^0 + 0 = 1$

$m = -\frac{1}{1+\ln 5}$

$y - 1 = -\frac{1}{1+\ln 5}(x - 0)$ or $y = 1 - \frac{x}{1+\ln 5}$

If $g(x) = \frac{(3x+5)^2}{\sqrt{x}}$, find $g''(x)$. **<u>SIMPLIFY YOUR ANSWER, AND FACTOR.</u>** **SCORE: _____ / 10 POINTS**

$g(x) = (9x^2 + 30x + 25)x^{-\frac{1}{2}} = 9x^{\frac{3}{2}} + 30x^{\frac{1}{2}} + 25x^{-\frac{1}{2}}$

$g'(x) = \frac{27}{2}x^{\frac{1}{2}} + 15x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}}$

$g''(x) = \frac{27}{4}x^{-\frac{1}{2}} - \frac{15}{2}x^{-\frac{3}{2}} + \frac{75}{4}x^{-\frac{5}{2}}$

$= \frac{3}{4}x^{-\frac{5}{2}}(9x^2 - 10x + 25)$

The resolution of each photo in your photo collection depends on the amount of storage you have for the collection. **SCORE: ___ / 8 POINTS**
If $r = f(s)$, where r is the resolution of each photo (in hundreds of dots per inch), and s is the storage available (in gigabytes), what does the statement $f'(9) = 0.5$ mean ? Give the units of measurement for each number in your answer.
<u>NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".</u>

IF YOU HAVE 9 GIGABYTES OF STORAGE AVAILABLE, THE RESOLUTION OF EACH PHOTO CAN INCREASE BY 50 DOTS PER INCREASE FOR EACH ADDITIONAL GIGABYTE OF STORAGE YOU ADD.

If $q(t) = (\sec t)^{\cot t}$, find $q'(t)$. **SCORE: _____ / 22 POINTS**

$\ln q(t) = \cot t \ln \sec t$

$\frac{q'(t)}{q(t)} = -\csc^2 t \ln \sec t + \cancel{\cot t} \frac{\cancel{\sec t}\, \cancel{\tan t}}{\cancel{\sec t}}$

$q'(t) = q(t)[-\csc^2 t \ln \sec t + 1]$

$= (\sec t)^{\cot t}(1 - \csc^2 t \ln \sec t)$

Show that $x^2 + y^2 = ky$ and $x^2 + y^2 = cx$ are orthogonal trajectories (where k and c are constants). **SCORE: _____ / 22 POINTS**

$$x^2+y^2=ky \qquad\qquad x^2+y^2=cx$$

$$2x+2y\frac{dy}{dx}=k\frac{dy}{dx} \qquad\qquad 2x+2y\frac{dy}{dx}=c$$

$$\frac{dy}{dx}=\frac{2x}{k-2y} \qquad\qquad \frac{dy}{dx}=\frac{c-2x}{2y}$$

$$\frac{\cancel{2}x}{k-2y}\cdot\frac{c-2x}{\cancel{2}y}=\frac{cx-2x^2}{ky-2y^2}=\frac{x^2+y^2-2x^2}{x^2+y^2-2y^2}$$

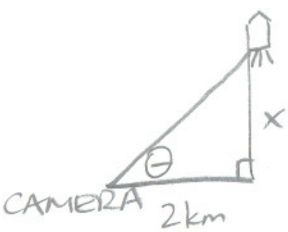
$$=\frac{y^2-x^2}{x^2-y^2}$$

$$=-1$$

State the definition of e from section 3.1 of the textbook (the definition used during the proof that $(e^x)' = e^x$.) **SCORE: _____ / 7 POINTS**

SEE VERSION A KEY

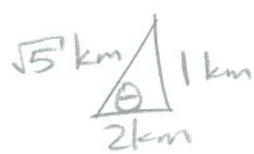
A television camera is positioned 2 km from the base of a rocket launching pad. The rocket rises vertically and its speed is 0.25 km per second when it has risen 1 km. If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that moment? **Specify the units for your final answer.** **SCORE: _____ / 22 POINTS**



$$\left.\frac{dx}{dt}\right|_{x=1\text{ km}}=0.25\text{ km/sec}$$

WANT $\frac{d\theta}{dt}$ WHEN $x=1$ km

$$\tan\theta=\frac{x}{2\text{ km}} \longrightarrow \text{WHEN } x=1\text{ km},$$



$\sec\theta=\frac{\sqrt{5}}{2}$

$$\sec^2\theta\frac{d\theta}{dt}=\frac{1}{2\text{ km}}\frac{dx}{dt}$$

$$\left(\frac{\sqrt{5}}{2}\right)^2\frac{d\theta}{dt}=\frac{1}{2\cancel{\text{km}}}\frac{0.25\cancel{\text{km}}}{\text{sec}}$$

$$\frac{d\theta}{dt}=\frac{1}{2}\,\frac{1}{4}\,\frac{4}{5}/\text{sec}=\frac{1}{10}\text{ RADIAN/SEC}$$

BONUS POINTS ON OTHER SIDE ➡