Math 1A
Midterm 2 Review Answers

[1] Using a linear approximation at $x=\frac{\pi}{6}$, $\csc 0.5 \approx \frac{6+(\pi-3)\sqrt{3}}{3}$

[2] $dx=0.5$, $\Delta y=-\frac{9}{100}$ and $dy=-\frac{1}{8}$

[3] $\sec x \tan^3 x + 5\sec^3 x \tan x$

[4] [a] $s'(1)=\frac{25}{2}$ [b] $s''(t)=\frac{3}{4}t^{-\frac{5}{2}}(10t^3+4t^2-3)$

[5] $y-9=12(x-2)$ and $y+7=12(x+2)$

[6] $y-4=7(x-2)$

[7] $\frac{1}{2}$

[8] [a] $y+8=12(x-2)$ [b] $y-\frac{1}{2}=-\frac{1}{4}(x+1)$ [c] $y+\frac{\pi}{4}=x+3$ [d] $y=-3(x-4)$

[9] **ANSWER WITHHELD (SHOW ME YOUR SOLUTION FOR VERIFICATION)**

[10] $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$

[11] $6xg'(x^2)+4x^3g''(x^2)$

[12] $y=4(x+1)$

[13] $4ax^3\left(-\frac{x}{4y}\right)=-\frac{ax^4}{y}=-\frac{y}{y}=-1$

[14] $\frac{dy}{dx}=(\sin x)^{\frac{1}{x}}\left(\frac{\cos x}{x\sin x}-\frac{\ln\sin x}{x^2}\right)=\frac{(x\cos x-\sin x\ln\sin x)(\sin x)^{\frac{1}{x}}}{x^2\sin x}=x^{-2}(x\cos x-\sin x\ln\sin x)(\sin x)^{\frac{1}{x}-1}$

[15] $f(x)=xe^{-x}$, $a=-1$ and $\lim_{h\to 0}\frac{(h-1)e^{1-h}+e}{h}=f'(-1)=(e^{-x}-xe^{-x})\Big|_{x=-1}=2e$

[16]
$$\begin{aligned}(\csc x)' &= \lim_{h\to 0}\frac{\csc(x+h)-\csc x}{h}\\ &= \lim_{h\to 0}\frac{\frac{1}{\sin(x+h)}-\frac{1}{\sin x}}{h}\\ &= \lim_{h\to 0}\frac{\sin x-\sin(x+h)}{h\sin(x+h)\sin x}\\ &= \lim_{h\to 0}\frac{\sin x-\sin x\cos h-\cos x\sin h}{h\sin(x+h)\sin x}\end{aligned}$$

$$
\begin{aligned}
&= \lim_{h\to 0} \frac{-\sin x\left(\dfrac{\cos h - 1}{h}\right) - \cos x\left(\dfrac{\sin h}{h}\right)}{\sin(x+h)\sin x} \\
&= \frac{-(\sin x)(0) - (\cos x)(1)}{\sin x \sin x} \\
&= \frac{-\cos x}{\sin x \sin x} \\
&= -\frac{1}{\sin x}\frac{\cos x}{\sin x} \\
&= -\csc x \cot x
\end{aligned}
$$