Math 1A (7:30am – 8:20am)
Midterm 3
Wed Dec 1, 2010

NAME YOU ASKED TO BE CALLED: ____________________

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK / USE PROPER NOTATION / SIMPLIFY YOUR ANSWERS

Rolle's Theorem applies to **only one** of the following situations. Find the one situation where Rolle's Theorem applies, and explain **very briefly** why Rolle's Theorem does not apply to all the other situations.

SCORE: _____ / 15 POINTS

SITUATION	WRITE "APPLIES" IF ROLLE'S THEOREM APPLIES OR EXPLAIN VERY BRIEFLY WHY ROLLE'S THEOREM DOES NOT APPLY
$f(x)=x^{\frac{5}{3}}$ on $[-1,1]$	$f(-1)=-1 \neq f(1)=1$
$f(x)=x^{\frac{4}{3}}$ on $[-1,1]$	APPLIES
$f(x)=x^{\frac{2}{3}}$ on $[-1,1]$	$f'(x)=\frac{2}{3}x^{-\frac{1}{3}}$ $f'(0)$ DNE (NOT DIFF.)
$f(x)=x^{-\frac{2}{3}}$ on $[-1,1]$	$f(0)$ DNE (NOT CONT.)

Find the following limits.

SCORE: _____ / 22 POINTS

[a] $\lim_{x\to0^+}(2-e^x)^{\csc x}$ 1^∞

$$\lim_{x\to0^+} \ln(2-e^x)^{\csc x}$$

$$= \lim_{x\to0^+} \csc x \ln(2-e^x) \quad \infty\cdot0$$

$$= \lim_{x\to0^+} \frac{\ln(2-e^x)}{\sin x} \quad \frac{0}{0}$$

$$= \lim_{x\to0^+} \frac{\frac{1}{2-e^x}\cdot -e^x}{\cos x}$$

$$= \frac{1\cdot(-1)}{1}$$

$$= -1$$

$$\lim_{x\to0^+}(2-e^x)^{\csc x} = e^{-1} = \frac{1}{e}$$

[b] $\lim_{x\to0^+}(2-x)^{\frac{1}{x}}$ 2^∞

$= \infty$

Let $f(x) = \dfrac{x^2 - x - 2}{x^2}$.

SCORE: _____ / 39 POINTS

[a] Find the domain of f.

$\{x \neq 0\}$

[b] Find the intercepts of f.

Y-INT : NONE - $f(0)$ DNE

X-INT : $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

[c] Find the discontinuities of f, and find both one-sided limits at each discontinuity. State the type of each discontinuity.

DISCONT @ $x=0$

$\lim\limits_{x\to 0^+} \dfrac{x^2-x-2}{x^2} = -\infty$ $\quad \frac{-2}{0^+}$

$\lim\limits_{x\to 0^-} \dfrac{x^2-x-2}{x^2} = -\infty$ $\quad \frac{-2}{0^+}$

INFINITE DISCONTINUITY / VERTICAL ASYMPTOTE @ $x=0$

[d] Find all horizontal asymptotes of f. **Use proper calculus notation.**

$\lim\limits_{x\to \infty} \dfrac{x^2-x-2}{x^2} = \lim\limits_{x\to\infty}\left(1 - \dfrac{1}{x} - \dfrac{2}{x^2}\right) = 1 - 0 - 0 = 1$

$\lim\limits_{x\to -\infty} \dfrac{x^2-x-2}{x^2} = \lim\limits_{x\to-\infty}\left(1 - \dfrac{1}{x} - \dfrac{2}{x^2}\right) = 1 - 0 - 0 = 1$

HORIZONTAL ASYMPTOTE @ $y=1$

OR

$\lim\limits_{x\to \infty} \dfrac{x^2-x-2}{x^2} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\infty} \dfrac{2x-1}{2x} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\infty} \dfrac{2}{2} = 1$

$\lim\limits_{x\to -\infty} \dfrac{x^2-x-2}{x^2} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to-\infty} \dfrac{2x-1}{2x} \overset{\frac{-\infty}{-\infty}}{=} \lim\limits_{x\to-\infty} \dfrac{2}{2} = 1$

THIS QUESTION CONTINUES ON THE NEXT PAGE ➡

[e] Graph f by finding all intervals where f is increasing/decreasing, all intervals where f is concave up/down, all local extrema, all inflection points, and all horizontal/vertical tangent lines. **Show proper calculus level work.**
Fill in the boxes at the bottom to check that you have completed all steps. Write "N/A" if not applicable.

$f(x) = 1 - x^{-1} - 2x^{-2}$

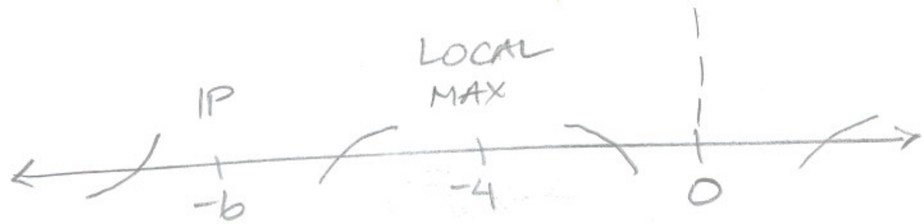
$f'(x) = x^{-2} + 4x^{-3} = x^{-3}(x+4)$

$f''(x) = -2x^{-3} - 12x^{-4} = -2x^{-4}(x+6)$

f', f'' UNDEFINED AT $x=0 \notin$ DOMAIN

$f' = 0$ @ $x = -4$

$f'' = 0$ @ $x = -6$

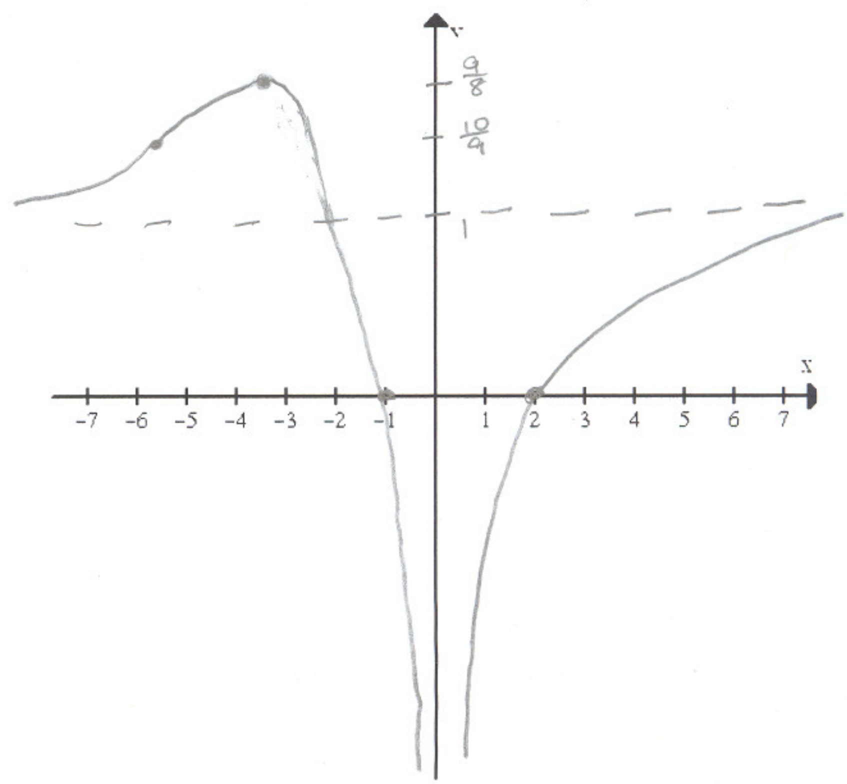


f'	+	+	−	+
f''	+	−	−	−

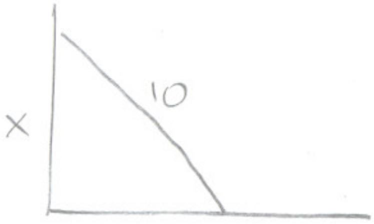
$f(-6) = \frac{10}{9}$ $\quad$ $f(-4) = \frac{9}{8}$

Intervals where f is increasing	$(-\infty,-4), (0,\infty)$
Intervals where f is decreasing	$(-4,0)$
Intervals where f is concave up	$(-\infty,-6)(0,\infty)$
Intervals where f is concave down	$(-6,0)$
Local maxima	-4
Local minima	NONE
Inflection points	-6
Horizontal tangent lines	$x=-4$
Vertical tangent lines	NONE

LABEL THE y-AXIS SO THAT THE CONCAVITY OF THE THE FUNCTION IS CLEARLY VISIBLE

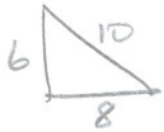


A 10 foot ladder is leaning against a wall. If the top of the ladder is sliding down the wall at 2 feet per second, **SCORE: _____ / 22 POINTS**
how quickly is the area under the ladder changing when the base of the ladder is 8 feet from the wall ?



$\frac{dx}{dt} = -2\,ft/s$

WANT $\frac{dA}{dt}$ WHEN $x = 6\,ft$



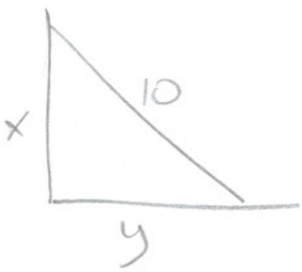
$$A = \frac{1}{2} x \sqrt{100 - x^2}$$

$$\frac{dA}{dt} = \left[\frac{1}{2}\left(\sqrt{100-x^2} + x \cdot \frac{1}{2}\frac{1}{\sqrt{100-x^2}} \cdot -2x\right)\right]\frac{dx}{dt}$$

$$\left.\frac{dA}{dt}\right|_{x=6ft} = \frac{1}{2}\left(8\,ft + 6\,ft \cdot \frac{1}{2} \cdot \frac{1}{8\,ft} \cdot -2(6\,ft)\right)\left(-2\frac{ft}{s}\right)$$

$$= -\frac{7}{2}\,ft^2/s$$

OR



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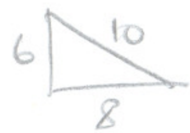


$$A = \frac{1}{2}xy \qquad x^2 + y^2 = 100$$

$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}y + x\frac{dy}{dt}\right) \qquad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$= \frac{1}{2}\left(\left(-2\frac{ft}{s}\right)(8\,ft) + (6\,ft)\left(\frac{3}{2}\frac{ft}{s}\right)\right) \qquad 2(6\,ft)\left(-2\frac{ft}{s}\right) + 2(8\,ft)\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{3}{2}\frac{ft}{s}$$

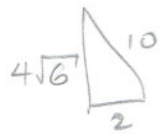
$$= -\frac{7}{2}\frac{ft}{s}$$

You want to lean a 10 foot ladder against a wall, so that the bottom of the ladder is at least 2 feet from the wall, and the top of the ladder is at least 6 feet from the ground. How should you lean the ladder so that the area under the ladder is as small as possible ? **You must use a calculus based argument.** **SCORE: _____ / 22 POINTS**

MINIMIZE A = AREA UNDER LADDER

CAN CHANGE X

$$A = \frac{1}{2} x \sqrt{100-x^2}$$



$$6 \le x \le 4\sqrt{6}$$

$$A' = \frac{1}{2}\left(\sqrt{100-x^2} - \frac{x^2}{\sqrt{100-x^2}}\right)$$

$$= \frac{1}{2} \cdot \frac{100-x^2-x^2}{\sqrt{100-x^2}}$$

$$= \frac{50-x^2}{\sqrt{100-x^2}}$$ IS UNDEFINED @ $x = \pm 10$ $\notin$ DOMAIN

$= 0$ IF $x = \pm 5\sqrt{2}$

$5\sqrt{2} \in$ DOMAIN

$A(6) = \frac{1}{2} 6 \cdot 8 = 24$

$A(4\sqrt{6}) = \frac{1}{2} 4\sqrt{6} \cdot 2 = 4\sqrt{6} < 4 \cdot 3 = 12$ ← GLOBAL MIN

$A(5\sqrt{2}) = \frac{1}{2} 5\sqrt{2} \cdot 5\sqrt{2} = 25$

$A = \frac{1}{2} y \sqrt{100-y^2}$ $\quad 2 \le y \le 8$

SAME A' AS OTHER METHOD

CRITICAL # $y = 5\sqrt{2}$

$A(2) = 4\sqrt{6}$ ← GLOBAL MIN

$A(8) = 24$

$A(5\sqrt{2}) = 25$

→ THE BASE OF THE LADDER SHOULD BE 2FT FROM THE WALL.

State Rolle's Theorem. **SCORE: _____ / 5 POINTS**

IF f IS CONTINUOUS ON $[a,b]$

AND DIFFERENTIABLE ON (a,b)

AND $f(a)=f(b)$

THEN FOR SOME $c \in (a,b)$, $f'(c)=0$

Estimate $\sqrt{0.9}$ using a linear approximation to $f(x)=\sqrt{1-x}$ at $x=0$. **SCORE: _____ / 15 POINTS**

<u>YOU MUST USE A LINEAR APPROXIMATION TO THE FUNCTION</u> $f(x)$ **<u>LISTED ABOVE.</u>**

$f'(x) = \frac{1}{2\sqrt{1-x}} \cdot -1$

$f'(0) = -\frac{1}{2}$ $\quad f(0)=1$

$L(x) = 1 - \frac{1}{2}(x-0) = 1 - \frac{1}{2}x$

$\sqrt{0.9} = \sqrt{1-0.1} = f(0.1) \approx L(0.1) = 1 - \frac{1}{2}(0.1)$

$= \frac{19}{20}$

Suppose f is a polynomial function such that $f''(x) = -56(x+8)^5(x+2)$. **SCORE: _____ / 10 POINTS**

If the critical numbers of f are -1 and -8, determine what the **<u>Second Derivative Test</u>** tells you about each critical number.

$f''(-1) = -56(7)^5(3) < 0$ LOCAL MAX

$f''(-8) = -56(0)^5(10) = 0$ NO CONCLUSION