

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED**SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION****YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS
UNLESS SPECIFICALLY ASKED FOR**

State the definition of "continuous (at a point)". Write in complete sentences, using proper English and algebra.

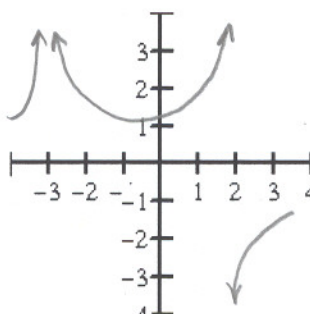
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f IS CONTINUOUS AT a IF $f(a)$ EXISTS,
 $\lim_{x \rightarrow a} f(x)$ EXISTS,
 AND $\lim_{x \rightarrow a} f(x) = f(a)$

Sketch the graph of a function f such that

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$$\begin{aligned}\lim_{x \rightarrow -3} f(x) &= \infty, \\ \lim_{x \rightarrow 2^+} f(x) &= -\infty \text{ and} \\ \lim_{x \rightarrow 2^-} f(x) &= \infty.\end{aligned}$$



$$\text{Let } f(x) = \begin{cases} 5-x, & \text{if } x < 1 \\ 2, & \text{if } x = 1 \\ x+3, & \text{if } 1 < x < 3 \\ 2x, & \text{if } x > 3 \end{cases}$$

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[a] Find $\lim_{x \rightarrow 2} f(x)$.

$$= \lim_{x \rightarrow 2} (x+3) = 5$$

[b] Find all values of x where f is not continuous. Justify your answers using the definition of continuity. **DO NOT USE A GRAPH.**

f IS NOT CONTINUOUS AT $x = 1$ SINCE $f(1)$ DNE

$$\left. \begin{aligned}\lim_{x \rightarrow 1^-} f(x) &= \lim_{x \rightarrow 1^-} (5-x) = 4 \\ \lim_{x \rightarrow 1^+} f(x) &= \lim_{x \rightarrow 1^+} (x+3) = 4\end{aligned} \right\} \text{ so } \lim_{x \rightarrow 1} f(x) = 4$$

$$\text{BUT } f(1) = 2 \neq \lim_{x \rightarrow 1} f(x)$$

so f IS NOT CONTINUOUS AT $x = 1$

CONTINUED ON OTHER SIDE

If $f(-3) = g(-3) = 5$ and $\lim_{x \rightarrow -3} f(x) = 4$ and $\lim_{x \rightarrow -3} g(x) = -1$, find $\lim_{x \rightarrow -3} [2f(x) - xg(x)]$.

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NOTE: Show proper use of the limit laws to justify your answer. You do NOT need to name the laws.

$$\begin{aligned} & 2 \lim_{x \rightarrow -3} f(x) - (\lim_{x \rightarrow -3} x)(\lim_{x \rightarrow -3} g(x)) \\ &= 2 \cdot 4 - (-3) \cdot (-1) \\ &= 5 \end{aligned}$$

State the definition of "vertical asymptote". Write in complete sentences, using proper English and algebra.

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f HAS A VERTICAL ASYMPTOTE AT $x=a$ IF
 $\lim_{x \rightarrow a^+} f(x) = \infty$ or $\lim_{x \rightarrow a^-} f(x) = \infty$ or $\lim_{x \rightarrow a^+} f(x) = -\infty$ or $\lim_{x \rightarrow a^-} f(x) = -\infty$

Find the following limits.

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[a] $\lim_{t \rightarrow -2} \frac{4t^2 - 3t - 12}{4 - 2t} = \frac{16 + 6 - 12}{4 + 4}$
 $= \frac{10}{8}$
 $= \frac{5}{4}$

[b] $\lim_{t \rightarrow 5} \frac{t-5}{3 - \sqrt{2t-1}} \cdot \frac{0}{0}$
 $= \lim_{t \rightarrow 5} \frac{t-5}{3 - \sqrt{2t-1}} \cdot \frac{3 + \sqrt{2t-1}}{3 + \sqrt{2t-1}}$
 $= \lim_{t \rightarrow 5} \frac{(t-5)(3 + \sqrt{2t-1})}{9 - (2t-1)}$
 $= \lim_{t \rightarrow 5} \frac{(t-5)(3 + \sqrt{2t-1})}{10 - 2t}$
 $= \lim_{t \rightarrow 5} - \frac{3 + \sqrt{2t-1}}{2}$
 $= - \frac{3+3}{2} = -3$

[c] $\lim_{t \rightarrow 1} \frac{\frac{3}{t+2} - \frac{1}{t}}{t-1} \cdot \frac{0}{0}$
 $= \lim_{t \rightarrow 1} \frac{\frac{3}{t+2} - \frac{1}{t}}{t-1} \cdot \frac{t(t+2)}{t(t+2)}$
 $= \lim_{t \rightarrow 1} \frac{3t - (t+2)}{t(t-1)(t+2)}$
 $= \lim_{t \rightarrow 1} \frac{2t - 2}{t(t-1)(t+2)}$
 $= \frac{2}{3}$