

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED**SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION****YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS
UNLESS SPECIFICALLY ASKED FOR**

State the definition of “vertical asymptote”. Write in complete sentences, using proper English and algebra.

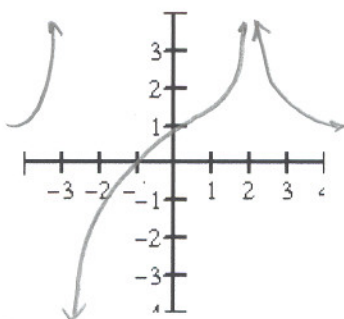
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SEE VERSION A KEY

Sketch the graph of a function f such that

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$$\begin{aligned}\lim_{x \rightarrow 2} f(x) &= \infty, \\ \lim_{x \rightarrow -3^+} f(x) &= -\infty \text{ and} \\ \lim_{x \rightarrow -3^-} f(x) &= \infty.\end{aligned}$$



$$\text{Let } f(x) = \begin{cases} 5-x, & \text{if } x < 1 \\ x+3, & \text{if } 1 < x < 3 \\ 4, & \text{if } x = 3 \\ 2x, & \text{if } x > 3 \end{cases}$$

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[a] Find $\lim_{x \rightarrow -2} f(x)$.

$$= \lim_{x \rightarrow -2} (5-x) = 7$$

[b] Find all values of x where f is not continuous. Justify your answers using the definition of continuity. **DO NOT USE A GRAPH.**f IS NOT CONTINUOUS AT $x = 1$ SINCE $f(1)$ DNE

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (x+3) = 6$$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} 2x = 6$$

$$\text{so } \lim_{x \rightarrow 3} f(x) = 6$$

$$\text{BUT } f(3) = 4 \neq \lim_{x \rightarrow 3} f(x)$$

so f IS NOT CONTINUOUS
AT $x = 3$

CONTINUED ON OTHER SIDE

$$\begin{aligned}
 \text{[a]} \quad \lim_{t \rightarrow 2} \frac{4t^2 - 3t - 12}{6 - 2t} &= \frac{16 - 6 - 12}{6 - 4} \\
 &= \frac{-2}{2} \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 \text{[b]} \quad \lim_{t \rightarrow 2} \frac{\frac{3}{t+4} - \frac{1}{t}}{t-2} &= \frac{0}{0} \\
 &= \lim_{t \rightarrow 2} \frac{\frac{3}{t+4} - \frac{1}{t}}{t-2} \cdot \frac{t(t+4)}{t(t+4)} \\
 &= \lim_{t \rightarrow 2} \frac{3t - (t+4)}{t(t-2)(t+4)} \\
 &= \lim_{t \rightarrow 2} \frac{2t - 4}{t(t-2)(t+4)} \\
 &= \frac{2}{2(6)} = \frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{[c]} \quad \lim_{t \rightarrow 3} \frac{t-3}{2 - \sqrt{3t-5}} &= \frac{0}{0} \\
 &= \lim_{t \rightarrow 3} \frac{t-3}{2 - \sqrt{3t-5}} \cdot \frac{2 + \sqrt{3t-5}}{2 + \sqrt{3t-5}} \\
 &= \lim_{t \rightarrow 3} \frac{(t-3)(2 + \sqrt{3t-5})}{4 - (3t-5)} \\
 &= \lim_{t \rightarrow 3} \frac{(t-3)(2 + \sqrt{3t-5})}{9 - 3t - 3} \\
 &= \frac{2+2}{-3} = -\frac{4}{3}
 \end{aligned}$$

If $f(-3) = g(-3) = 5$ and $\lim_{x \rightarrow -3} f(x) = 4$ and $\lim_{x \rightarrow -3} g(x) = -1$, find $\lim_{x \rightarrow -3} [xf(x) - 2g(x)]$.

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NOTE: Show proper use of the limit laws to justify your answer. You do NOT need to name the laws.

$$\begin{aligned}
 &= \left(\lim_{x \rightarrow -3} x \right) \left(\lim_{x \rightarrow -3} f(x) \right) - 2 \lim_{x \rightarrow -3} g(x) \\
 &= (-3)4 - 2(-1) \\
 &= -12 + 2 \\
 &= -10
 \end{aligned}$$

State the definition of "continuous (at a point)". Write in complete sentences, using proper English and algebra.

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SEE VERSION A KEY