

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED**SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION****YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS
UNLESS SPECIFICALLY ASKED FOR**

$$\text{Let } f(x) = \begin{cases} 5+x, & \text{if } x < -1 \\ 3-x, & \text{if } -1 < x < 2 \\ 4, & \text{if } x = 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$$

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[a] Find $\lim_{x \rightarrow 3} f(x)$.

$$= \lim_{x \rightarrow 3} (2x-3) = 3$$

[b] Find all values of x where f is not continuous. Justify your answers using the definition of continuity. **DO NOT USE A GRAPH.**

f IS NOT CONTINUOUS AT $x = -1$ SINCE $f(-1)$ DNE

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (3-x) = 1 \quad \left. \vphantom{\lim_{x \rightarrow 2^-} f(x)} \right\} \text{SO } \lim_{x \rightarrow 2} f(x) = 1$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (2x-3) = 1$$

$$\text{BUT } f(2) = 4 \neq \lim_{x \rightarrow 2} f(x)$$

SO f IS NOT CONTINUOUS
AT $x = 2$

State the definition of "vertical asymptote". Write in complete sentences, using proper English and algebra.

SCORE: ____ / 2 POINTS

SEE VERSION A KEY

If $f(-2) = g(-2) = -1$ and $\lim_{x \rightarrow -2} f(x) = 5$ and $\lim_{x \rightarrow -2} g(x) = 4$, find $\lim_{x \rightarrow -2} [xg(x) - 3f(x)]$.

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NOTE: Show proper use of the limit laws to justify your answer. You do NOT need to name the laws.

$$= \left(\lim_{x \rightarrow -2} x \right) \left(\lim_{x \rightarrow -2} g(x) \right) - 3 \lim_{x \rightarrow -2} f(x)$$

$$= (-2)(4) - 3(5)$$

$$= -23$$

CONTINUED ON OTHER SIDE

Find the following limits.

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[a] $\lim_{t \rightarrow -1} \frac{4t^2 - 3t - 12}{3 - 3t} = \frac{4 + 3 - 12}{3 + 3} = \frac{-5}{6}$

[b] $\lim_{t \rightarrow -1} \frac{\frac{3}{t-2} - \frac{1}{t}}{1+t} = \frac{0}{0}$

$$= \lim_{t \rightarrow -1} \frac{\frac{3}{t-2} - \frac{1}{t}}{1+t} \cdot \frac{t(t-2)}{t(t-2)}$$

$$= \lim_{t \rightarrow -1} \frac{3t - (t-2)}{t(t+1)(t-2)}$$

$$= \lim_{t \rightarrow -1} \frac{2t + 2}{t(t+1)(t-2)}$$

$$= \frac{2}{-1(-3)} = \frac{2}{3}$$

[c] $\lim_{t \rightarrow 2} \frac{t-2}{3 - \sqrt{4t+1}} = \frac{0}{0}$

$$= \lim_{t \rightarrow 2} \frac{t-2}{3 - \sqrt{4t+1}} \cdot \frac{3 + \sqrt{4t+1}}{3 + \sqrt{4t+1}}$$

$$= \lim_{t \rightarrow 2} \frac{(t-2)(3 + \sqrt{4t+1})}{9 - (4t+1)}$$

$$= \lim_{t \rightarrow 2} \frac{(t-2)(3 + \sqrt{4t+1})}{8 - 4t - 4}$$

$$= \frac{3+3}{-4} = -\frac{3}{2}$$

State the definition of "continuous (at a point)". Write in complete sentences, using proper English and algebra.

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SEE VERSION A KEY

Sketch the graph of a function f such that

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$$\lim_{x \rightarrow 2} f(x) = -\infty,$$

$$\lim_{x \rightarrow -3^+} f(x) = \infty \text{ and}$$

$$\lim_{x \rightarrow -3^-} f(x) = -\infty.$$

