

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED**SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION****YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS
UNLESS SPECIFICALLY ASKED FOR**

$$\text{Let } f(x) = \begin{cases} 3+x, & \text{if } x < -1 \\ 4, & \text{if } x = -1 \\ 1-x, & \text{if } -1 < x < 2 \\ 2x-5, & \text{if } x > 2 \end{cases}$$

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[a] Find $\lim_{x \rightarrow 1} f(x)$.

$$= \lim_{x \rightarrow 1} (1-x) = 0$$

[b] Find all values of x where f is not continuous. Justify your answers using the definition of continuity. **DO NOT USE A GRAPH.** f IS NOT CONTINUOUS AT $x = 2$ SINCE $f(2)$ DNE

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (3+x) = 2 \quad \left. \vphantom{\lim_{x \rightarrow -1^-} f(x)} \right\} \text{so } \lim_{x \rightarrow -1} f(x) = 2$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (1-x) = 2 \quad \text{BUT } f(-1) = 4 \neq \lim_{x \rightarrow -1} f(x)$$

SO f IS NOT CONTINUOUS AT $x = -1$

State the definition of "continuous (at a point)". Write in complete sentences, using proper English and algebra.

SCORE: ____ / 2 POINTS

SEE VERSION A KEY

If $f(-2) = g(-2) = -1$ and $\lim_{x \rightarrow -2} f(x) = 5$ and $\lim_{x \rightarrow -2} g(x) = 4$, find $\lim_{x \rightarrow -2} [2g(x) - xf(x)]$.

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NOTE: Show proper use of the limit laws to justify your answer. You do NOT need to name the laws.

$$= 2 \lim_{x \rightarrow -2} g(x) - \left(\lim_{x \rightarrow -2} x \right) \left(\lim_{x \rightarrow -2} f(x) \right)$$

$$= 2(4) - (-2)(5)$$

$$= 18$$

CONTINUED ON OTHER SIDE

State the definition of "vertical asymptote". Write in complete sentences, using proper English and algebra.

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SEE VERSION A KEY

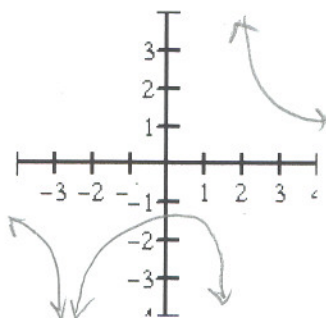
Sketch the graph of a function f such that

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$$\lim_{x \rightarrow -3} f(x) = -\infty,$$

$$\lim_{x \rightarrow 2^+} f(x) = \infty \text{ and}$$

$$\lim_{x \rightarrow 2^-} f(x) = -\infty.$$



Find the following limits.

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$$\begin{aligned} \text{[a]} \quad \lim_{t \rightarrow 2} \frac{4t^2 - 3t - 12}{4 + 2t} &= \frac{16 - 6 - 12}{4 + 4} \\ &= \frac{-2}{8} \\ &= -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{[b]} \quad \lim_{t \rightarrow -1} \frac{t+1}{2 - \sqrt{2t+6}} &= \frac{0}{0} \\ &= \lim_{t \rightarrow -1} \frac{t+1}{2 - \sqrt{2t+6}} \cdot \frac{2 + \sqrt{2t+6}}{2 + \sqrt{2t+6}} \\ &= \lim_{t \rightarrow -1} \frac{(t+1)(2 + \sqrt{2t+6})}{4 - (2t+6)} \\ &= \lim_{t \rightarrow -1} \frac{(t+1)(2 + \sqrt{2t+6})}{-2t - 2} \\ &= \frac{2+2}{-2} = -2 \end{aligned}$$

$$\begin{aligned} \text{[c]} \quad \lim_{t \rightarrow -2} \frac{\frac{3}{t-4} - \frac{1}{t}}{2+t} &= \frac{0}{0} \\ &= \lim_{t \rightarrow -2} \frac{\frac{3}{t-4} - \frac{1}{t}}{2+t} \cdot \frac{t(t-4)}{t(t-4)} \\ &= \lim_{t \rightarrow -2} \frac{3t - (t-4)}{t(t+2)(t-4)} \\ &= \lim_{t \rightarrow -2} \frac{2t+4}{t(t+2)(t-4)} \\ &= \frac{2}{-2(-6)} = \frac{1}{6} \end{aligned}$$