Math 1A (7:30am – 8:20am)
Quiz 3 Version A
Fri Oct 8, 2010

NAME YOU ASKED TO BE CALLED: _____

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER CALCULUS-LEVEL ALGEBRAIC WORK
USE PROPER NOTATION

State the definition of "horizontal asymptote". Write in complete sentences, using proper English and algebra. **SCORE: ___ / 2 POINTS**

f HAS A HORIZONTAL ASYMPTOTE AT $y=b$
IF $\lim_{x\to\infty} f(x)=b$ OR $\lim_{x\to-\infty} f(x)=b$

State the Squeeze Theorem. Write in complete sentences, using proper English and algebra. **SCORE: ___ / 2 POINTS**

IF $f(x)\le g(x)\le h(x)$ FOR ALL x NEAR a (BUT POSSIBLY NOT AT a)
AND $\lim_{x\to a} f(x)=\lim_{x\to a} h(x)=L$, THEN $\lim_{x\to a} g(x)=L$

Prove that the equation $x^4 = x+3$ has a solution in the interval $(0, 2)$. Justify your reasoning properly. **SCORE: ___ / 5 POINTS**

LET $f(x)=x^4-x-3$
f IS CONT. ON $[0,2]$ SINCE f IS A POLYNOMIAL
$f(0)=-3$ AND $f(2)=11$, SO $f(0)<0<f(2)$
BY IVT, THERE IS A $c\in(0,2)$ SUCH THAT $f(c)=0$
IE. $c^4-c-3=0$
$c^4=c+3$

The time it takes to run a certain distance depends on the speed you run. If $t = f(v)$, where t is the time (in minutes), and v is your speed (in miles per hour), what does the statement $f'(6) = -8$ mean ? Give the units of measurement for each number in your answer. **SCORE: ___ / 2 POINTS**

NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

IF YOU RUN AT 6 MPH,
YOUR TIME WILL DECREASE BY 8 MINUTES
FOR EACH MPH YOU RUN FASTER.

Fill in each blank with either a number, ∞, $-\infty$ or DNE (write DNE if no other answer is appropriate). **SCORE: ___ / 3 POINTS**

NOTE: You do NOT need to show work.

[a] $\lim_{x\to\infty} \tan x =$ DNE

[b] $\lim_{x\to-\infty} 3^{-x} =$ ∞

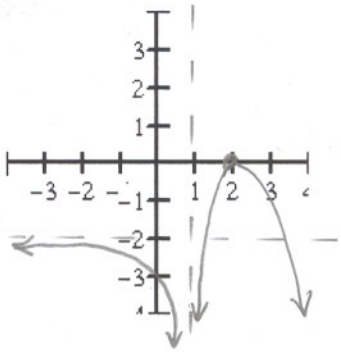
[c] $\lim_{x\to-\infty} (1-x^3) =$ ∞

CONTINUED ON OTHER SIDE

Sketch the graph of a function f such that

SCORE: ___ / 3 POINTS

f is continuous everywhere except at $x = 1$,
$f(2) = 0$,
$\lim_{x \to 1} f(x) = -\infty$,
$\lim_{x \to \infty} f(x) = -\infty$ and
$\lim_{x \to -\infty} f(x) = -2$.



Find $\lim_{x \to -\infty} \dfrac{7+x}{\sqrt{5+4x^2}}$.

SCORE: ___ / 5 POINTS

$$= \lim_{x \to -\infty} \frac{7+x}{\sqrt{5+4x^2}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x \to -\infty} \frac{\frac{7}{x}+1}{\sqrt{5+4x^2} \cdot -\sqrt{\frac{1}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{\frac{7}{x}+1}{-\sqrt{\frac{5}{x^2}+4}}$$

$$= \frac{0+1}{-\sqrt{0+4}} = -\frac{1}{2}$$

Let $f(x) = -\dfrac{1}{x^2}$.

SCORE: ___ / 8 POINTS

[a] Find $f'(x)$. **NOTE: If you have taken calculus before, do NOT use differentiation shortcuts.**

$$f'(x) = \lim_{h \to 0} \frac{-\frac{1}{(x+h)^2} - -\frac{1}{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{-x^2+(x+h)^2}{h(x+h)^2x^2}$$

$$= \lim_{h \to 0} \frac{2xh+h^2}{h(x+h)^2x^2}$$

$$= \lim_{h \to 0} \frac{2x+h}{(x+h)^2x^2}$$

$$= \frac{2x}{x^4}$$

$$= \frac{2}{x^3}$$

[b] Find the **equation** of the tangent line to the graph of $y = f(x)$ at $x = 3$.

$$f'(3) = \frac{2}{27}$$

$$y - \left(-\frac{1}{9}\right) = \frac{2}{27}(x-3)$$

$$y + \frac{1}{9} = \frac{2}{27}(x-3) \quad \text{OR} \quad y = -\frac{1}{9} + \frac{2}{27}(x-3)$$

$$\text{OR} \quad y = \frac{2}{27}x - \frac{1}{3}$$